FACULTY RESEARCH EDITION

of

The Savannah State College Bulletin

Published by

The Savannah State College

Volume 21, No. 2 Savannah, Georgia December, 1967

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On Strengths of Shock Waves with Respect to Thermodynamic Parameters

by

Nazir A. Warsi

1. INTRODUCTION

If F is thermodynamic parameter, then the F-strength of a shock wave is given by

(1.1) $\delta_{F/} = [F]/F_{1}$

Hence, we define enthalpy, sound velocity, internal energy, temperature, entropy, and obliquity strengths of the shock wave by

(1.2) $\delta_{I/} = [I]/I_{1/}$,

(1.3) $\delta_{c^2/} = [c^2]/c^2_{1/}$,

(1.4) $\delta e = [e]/e_{1/}$,

(1.5) $\delta_{T/} = [T]/T_{1/}$,

(1.6) $\delta_{\eta/} = [\eta]/\eta_{1/}$,

and

(1.7) $\psi_{2/\alpha} = (1 + \delta_{\psi/})\psi_{1/\alpha}$.
respectively.

2. SHOCK STRENGTHS WITH RESPECT TO THERMODYNAMIC PARAMETERS

We have the following theorems in this connection.

Theorem 2.1: *$\delta_{\gamma/}$ and $\delta_{I/}$ are related by*

$$(2.1) \quad \delta_{I/} = \frac{-(\gamma-1)\, h^2_{n/}\, \delta_{\gamma/}\, (\delta_{\gamma/} + 2)\, \gamma_{1/}}{2\gamma\, p_{1/}}$$

or

$$(2.2) \quad \delta_{I/} = \frac{\gamma - 1}{2\gamma p}\, h^2_{n/}\, \delta_{\gamma/} (\delta_{\gamma/} + 2)\, U_{1n/} .$$

Proof: The specific enthalpy behind a shock surface is given by [3]

$$(2.3) \quad [I] = -\frac{1}{2} \dot{h}^2_{n/}\, \delta_{\gamma/} (\delta_{\gamma/} + 2)\, \gamma^2_{1/} .$$

which, with the help of (1.2) and the relation $I_{1/} = \frac{\gamma}{\gamma - 1} p_{1/} \gamma_{1/}$,

gives (2.1). The equation (2.2) is easily obtained from (2.1) and the relation

$-\dot{h}_{n/} \gamma_{1/} = U_{1n/}$.

Theorem 2.2: *The value of* $\delta_{I/}$ *is given by*

$$(2.4) \quad \delta_{1/} = \frac{-2(\gamma - 1)\, \dot{h}_{n/} (C_{1/}^2 - \dot{h}_{1/}^2 \gamma_{1/}^2)(C_{1/}^2 + \gamma \dot{h}_{n/}^2 \gamma_{1/}^2)}{\gamma (\gamma + 1)^2 p_{1/} \dot{h}_{n/}^3 \gamma_{1/}^3}$$

Proof: We know that [1]

$$(2.5) \quad \delta_{\gamma/} = \frac{2(C_{1/}^2 - \dot{h}_{n/}^2 \gamma_{1/}^2)}{(\gamma + 1)\, \dot{h}_{n/}^2 \gamma_{1/}^2}.$$

On substituting from (2.5) in (2.1), we get (2.4).

Theorem 2.3: *The sound velocity strength is the same as the specific enthalpy strength.*

Proof: For a polytropic gas, we have

$$(2.6) \quad I_{\alpha/} = \frac{\gamma}{\gamma - 1} C_{\alpha/}^2 .$$

which easily gives

$$(2.7) \quad [I] = \frac{\gamma}{\gamma - 1} [C^2]$$

Equations (1.2) and (1.3) reduce (2.7) to

$$(2.8) \quad \delta_{I/} I_{1/} = \frac{\gamma}{\tau - 1} \delta c^2_{/} C_{1/}^2 .$$

Now, (2.6) and (2.8) give

$$(2.9) \quad \delta_{I/} = \delta_{c/}^2 .$$

Theorem 2.4: *The internal energy and specific enthalpy strengths are the same.*

Proof: For a polytropic gas, we have

$$(2.10) \quad I_{\alpha/} = \frac{1}{\gamma} e_{\alpha/} .$$

This gives

$$(2.11) \quad [I] = \frac{1}{\gamma} [e]$$

On substituting the values of [I] and [e] from (1.2) and (1.4) respectively, (2.11) becomes

$$(2.12) \quad I_{1/} \delta_{I/} = \delta_{e/} \frac{e_{1/}}{\gamma}.$$

This, in virtue of (2.10), gives

$$(2.13) \quad \delta_{I/} = \delta_{e/}.$$

Theorem 2.5: *The temperature and specific enthalpy shock strengths are the same.*

Proof: For a polytropic gas, we have

$$(2.14) \quad I_{\alpha/} = \frac{\gamma}{\gamma - 1} R\, T_{\alpha/}.$$

which gives

$$(2.15) \quad [I] = \frac{\gamma}{\gamma - 1} R\, [T]$$

This equation, in virtue of (1.2), (1.5), and (2.14) becomes

$$(2.16) \quad \delta_{I/} = \delta_{T/}$$

Theorem 2.6: *The obliquity strength can be determined from*

$$(2.17) \quad \delta_{\psi/} = \frac{\dot{h}_{n/} \gamma_{1/} \delta_{\gamma/}}{U_{1n/} - \dot{h}_{n/} \gamma_{1/} \delta_{\gamma/}}.$$

or

$$(2.18) \quad \delta_{\psi/} = \frac{- V_{in/} \delta_{\gamma/}}{U_{1n/} + V_{1n/} \delta_{\gamma/}}.$$

Proof: The equation (1.5) gives

$$(2.19) \quad \psi_{2/\alpha} = (\delta_{\psi/} + 1)\, \psi_{1/\alpha}.$$

Also, the components of obliquity in the region $\beta/$ is given by [2]

$$(2.20) \quad \psi_{\beta/\alpha} = U_{\beta/i} X^{i}_{,\alpha} / U_{\beta n/}.$$

which, for the region 2/, becomes

$$(2.21) \quad \psi_{2/\alpha} = U_{2/i} X^{i}_{,\alpha} / U_{2n/}.$$

Now, the normal velocity behind the shock surface is given [1]

$$(2.22) \quad [U_n] = \delta_{\gamma/} V_{1n/} .$$

Hence, (2.17) can be easily deduced from (2.19), (2.20), (2.21), and (2.22) by simple substitution. Equations (2.17) gives (2.18) with the help of the relation $-\dot{h}_{n/}\gamma_{1/} = V_{1n/}$.

Theorem 2.7: The entropy strength of the shock wave is given by

$$(2.23) \quad \delta_{\eta/} = \log \frac{(\delta\gamma + 1)^{\gamma-1}}{2p_{1/}\gamma_{1/}^{\gamma} c_{1/}^{2}} \left\{ 2c_{1/}^{2} - (\gamma - 1)\, \delta_{\gamma/} (\delta_{\gamma/} + 2)\, \dot{h}_{n/}^{2} \gamma_{1/}^{2} \right\}$$

Proof: The entropy behind a shock surface can be determined from [3]

$$(2.24) \quad [\eta] = JC_U \log \frac{(\delta_{\gamma/})^{\gamma-1}}{2\, c_{1/}^{2}} \left\{ 2c_{1/}^{2} - (\gamma - 1)\, \delta_{\gamma/} (\delta_{\gamma/} + 2)\, \dot{h}_{n/}^{2} \gamma_{1/}^{2} \right\}$$

This equation gives (2.23) on substitution from (1.6).

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On Shock Strengths with Respect To Flow Parameters

by

Nazir A. Warsi

1. INTRODUCTION

The shock strength with respect to the specifiic volume γ is given by

(1.1) $\delta_{\gamma /} = [\gamma]/\gamma_{1/}$

Similarly, the velocity and pressure shock strengths can be given by

(1.2) $\delta u_{n/} = [u_{n/}]/u_{1n/}$

and

(1.3) $\delta_{p/} = [p]/p$.

In what follows, an attempt has been made to find the relations between different parametric shock strengths.

2. DIFFERENT SHOCK STRENGTHS

We have the following theorems in this regard.

Theorem 2.1: *For a given $\delta_{\gamma /}$, the velocity shock strength is given by*

(2.1) $\delta u_{n/} = - \delta_{\gamma /} \dot{h}_{n/} \gamma_{1/}$,

or

(2.2) $\delta u_{n/} = \delta_{\gamma /} U_{1n/}/u_{1n/}$,

which, for a stationary shock, reduces to

(2.3) $\delta u_{n/} = \delta_{\gamma /}$

Proof: The normal velocity behind the shock surface is given by [1]

(2.4) $[u_{n/}] = - \dot{h}_{n/} \delta_{\gamma} \gamma_{1/}$,

or

(2.5) $[u_{n/}] = \delta_{\gamma /} v_{1n/}$

Equations (2.4) and (2.5) together with (1.2) give (2.1) and (2.2) respectively. For a stationary shock surface, $v_{1n/} = u_{1n/}$. Thus, (2.2) reduces to (2.3).

Theorem 2.2: *The velocity strength of the shock is given by*

$$(2.6)\quad \delta_{un/} = \frac{-2}{\gamma + 1}, \frac{c_{1/}^{2} - \dot{h}_{n/}^{2}\gamma_{1/}}{u_{1/n} - \dot{h}_{n/}\nu_{1/}},$$

or

$$(2.7)\quad \delta_{un/} = \frac{-2}{\gamma + 1}, \frac{\gamma p_{1/} - \dot{h}_{n/}^{2}\gamma_{1/}}{\dot{h}_{n/} u_{1n/}}.$$

or

$$(2.8)\quad \delta_{un/} = \frac{-2}{\gamma + 1}, \frac{c_{1/}^{2} - v_{1n/}^{2}}{v_{1n/} u_{1n/}},$$

or

$$(2.9)\quad \delta_{un/} = \frac{2}{\gamma + 1}, \frac{\gamma p_{1/} + \dot{h}_{n/} v_{1n/}}{\dot{h}_{n/} u_{1n/}}.$$

Proof: For the value of $\delta_{\gamma/}$ we have [2]

$$(2.10)\quad \delta_{\gamma/} = \frac{2 \qquad (c_{1/}^{2} - \dot{h}_{n/}^{2}\gamma_{1/}^{2}),}{(\gamma + 1) \qquad \dot{h}_{n/}^{2}\gamma_{1/}^{2}}$$

or

$$(2.11)\quad \delta_{\gamma 1} = \frac{2\,(\gamma p_{1/} - \dot{h}_{n/}^{2}\gamma_{1/})}{\gamma_{1/}\dot{h}_{n/}^{2}}$$

Substituting from these equations in (2.1) and (2.2), we get (2.6) and (2.7). Now, the relation $-\dot{h}_{n/}\gamma_{1/} = v_{1n/}$ reduces (2.6) and (2.7) to (2.8) and (2.9) respectively.

Theorem 2.3: *The pressure strength* $\delta_{p/}$ *is related to* $\delta_{\gamma/}$ *by*

$$(2.12)\quad \delta_{p/} = -\dot{h}_{n/}^{2}\gamma_{1/}\delta_{\gamma 1} / p_{1/}$$

or

$$(2.13)\quad \delta_{p/} = \dot{h}_{n/} u_{1n/}\delta_{\gamma/}.$$

which, for a stationary shock reduces to

$$(2.14)\quad \delta_{p/} = \dot{h}_{n/} u_{1n/}\delta_{\gamma 1} / p_{1/}.$$

Proof: The pressure behind the shock surface is given by [1]

$$(2.15) \quad [p] = h_{n/} \delta_{\gamma/} \gamma_{1/},$$

or

$$(2.16) \quad [p] = -\dot{h}_{n/} \delta_{\gamma/} v_{1n/}.$$

On substitution for [p] from (1.3), the equations (2.15) and (2.16) give (2.12) and (2.13) respectively. For a stationary shock, $v_1{}^n/ = u_1{}^n/$. Hence, (2.13) gives (2.14) on application of this condition.

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