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HOWARD JORDAN, JR., *President*

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Teaching Counting And The Fundamental Operations To Elementary School Teachers

by

Prince A. Jackson, Jr.

Instructors of mathematics for elementary school teachers have found through the years that their students can count[1] but do not know the principles of counting. While most elementary school teachers can count "ad infinitum," very few can really explain why 100 is-the next integer after 99. Indubitably, most elementary school teachers rely heavily on memory when they count. As a result, elementary school children have suffered because they often are taught how to count without any ideas of what they are doing.

The purpose of this paper is to describe a method of teaching counting to elementary school teachers that will minimize the role of rote in the process of counting. The method for using counting in the operations of addition and multiplication will be discussed. To build as strong a case as possible for the teaching method, the writer will utilize abstract symbols rather than the usual Hindu-Arabic symbols. The rationale for utilization of abstract symbols is that a student who learns to count with abstract symbols can handle the Hindu-Arabic symbols with great facility. It should be noted also that the concept of "base" can be discovered when the principles of counting are learned.

Let the symbols *, $, #, % and & be the digits of a numeration system. Let it be understood that the order of the digits are *, $, #, % and &. The student begins to learn how to count by merely repeating in the given order the names assigned the symbols above. Thus the student writes *, $, #, %, and &.[2] The next step is crucial in that it is the basic principle of counting. It is how the successor (the first double digit numeral) of the last single digit numeral is formed.

Since * has been used as the first digit in writing the first $ * numerals[3] of the system, the next digit in order after * is used as the first digit and * as the second digit of the numeral that succeeds &. Thus the student writes $*. He can easily see that the next numeral after $* is $$. Continuing in this manner, he reaches $&. Since $ has been used as the first digit in the series of numerals succeeding &, the student will have no difficulty in realizing that another digit must now be used as the first digit in the numeral that succeeds $&. By now, he realizes that the first digit in the successor of $& is # and should write #* as the successor of $&.

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[1]Counting can be defined as naming consecutively, a set of numbers in order of their size.

[2]It is vitally important that the student understands that the numerals can be written, **, *$, *#, *%, and *&. In ordinary counting, the first ten integers are 00, 01, 02, 03, 04, 05, 06, 07, 08, and 09.

[3]$* is a better symbol to use than 5 since it is undesirable for any notions of "base" to enter at this time.

To test understanding of this principle, the instructor might ask the student to write the last double digit numeral of the system. If the student fails to write && after a brief consideration of the question, the instructor should review the basic principle of counting.

To write the successor of &&, the student must recognize that it is a three-digit numeral. On the basis of the fundamental priniciple he realizes that $ succeeds & as the first numeral and * replaces & in the second place. Thus * is the third place digit since it is always the beginning digit in counting. Hence, it should be clear to the student that $** is the first three-digit numeral.

The student should be allowed to count through $*&. If he has really learned the basic principle of counting he will have no difficulty in writing $$* as the successor of $*&. If he can carry out this procedure, he will recognize &&& as the last three-digit numeral and $*** as the first four-digit numeral. By now, he is able to count and knows how to form the successor of any numeral in the system.

To teach addition, the instructor should have his students to construct a table similar to the following:

+	*	$	#	%	&
*	*	$	#	%	&
$	$	#	%	&	$*
#	#	%	&	$*	$$
%	%	&	$*	$$	$#
&	&	$*	$$	$#	$%

TABLE I

It is easy to lead the student to discover that the rows or columns can be completed by merely counting. Using Table I, he can learn the addition facts of the numeration system.[4] For example, he sees that %+& is $#. The student should study the table and its construction thoroughly.

Once he is familiar with the table, the student can now proceed to more difficult problems of addition. For example, he should attack addition problems such as $ #+ %. He can solve this problem as follows:

```
(Carry)   $
          $ #
            %
        -------
          # *
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Note that "carrying" is used here as it is used in familiar addition.

[4]It is not necessary for the student to memorize these addition facts. However, he must recognize * as the zero and $ as the unit of the system.

Other problems for example are (&&) + (%$) and (&$%) + (%#$). The student can solve these as follows:

(Carry)		$						
		&	&					
		#	%			&	$	%
		%	$			%	#	$
	#	*	%		$	#	%	&

Adding & and %, the sum is $#. When $ is added to this sum, the new sum is $%. Writing % under the second column, we can begin to add the first column with $ carried from the last sum. Adding $ and &, the sum is $*. Adding # to this sum the new sum is $#. When % is added to the latter sum, the final sum is #*. The second problem is rather easily handled and will not be discussed here.

To sharpen the skills of the students, the instructor should supply adequate problems for this purpose.

Once the students have mastered addition with the use of the table, they are ready to develop skills in multiplication in the system of numeration. Since the symbols are abstract, the students will have to construct a multiplication table. The multiplication table should be constructed by using the addition table. The multiplication table is similar to the following:

×	$	#	%	&
$	$	#	%	&
#	#	&	$$	$%
%	%	$$	$&	##
&	&	$%	##	%$

TABLE II

It is easy to lead the student to discover that the rows or columns can be completed by addition. For example, the #th row can be completed in the following manner. To get the first entry, simply write # since it is # × $.[5] To get the second entry, add # to # and get & from the addition table. To get the third entry, add # to & and get $$ from the addition table. To get the final entry of the row, add # to $$ and get $% by use of the addition table. Any column may be completed in an analogous manner. As an exercise to strengthen the counting skill of the students, the instructor could have them to complete the tables by counting. To complete the %th row, simply count by %'s.[6]

[5] $ is the unit of the system. By the identity property, $X any numeral of the system = that numeral.

[6] To count by %'s, write the numerals of the system in ascending order. The order is *, $, #, %, &, $, *, $, $, $$#, $%, $&, #*, #$, ##, #%, #&, . . . counting by %'s would yield the sequence %, $$, $&, ##. . .

Using Table II, the students can learn to work with the multiplication facts of numeration system.[7] For example it is obvious that % × & is # #. The student should study the table and its construction intensively since many problems of multiplication require the use of the addition table:

A typical exercise that the instructor might assign to the students is ($ #) × (# % &). This is carried out in the usual manner:

	#	%	&
		$	#
$	*	#	%
#	%	&	
%	&	$	%

Many more exercises of this kind will sharpen the students' skills in multiplication.

Since subtraction is the inverse operation of addition and division is the inverse operation of multiplication, the instructor can lead the students to discover how they can perform these operations by use of the tables. Subtraction is defined in the usual way. That is, to find m—n is to find a number p such that n + p is m. Using this definition, the student can solve problems similar to and check the answers by addition.

#	&	%	$
	&	&	#
$	&	%	&

Division is defined in the usual way. That is, to find m/n is to find p such that np is m. Using this definition, the student can solve problems similar to and check the answers by multiplication.

			%	&
$#)	$	*	$	%
		&	$	
		$	*	%
		$	*	%

The concept of "base" can now be developed. This is done by using the regular digits, 0, 1, 2, 3, and 4. The students will have no difficulty in constructing the addition and multiplication tables. By setting up a one-to-one correspondence between the regular numerals and the abstract numerals, the students will discover that the abstract system is really no different from the regular quinary arithemetic. At this point, other bases may be introduced with little or no real difficulty.

[7]It is not necessary for the student to memorize these facts.

In conclusion, the writer wishes to impress upon the reader that the above method of teaching counting with the use of an abstract system of numeration with no reference to "base" seems to produce superior results when compared with the method using the integers and base ten. The abstract system seems to produce better comprehension of the operations of addition and multiplication. As a result of this better comprehension, elementary school teachers can do a better job of teaching counting, addition, multiplication, subtraction, and division to their pupils.