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The Pedagogical Interrelationship Between Mathematics and Science

by

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The purpose of this paper is to examine the pedagogical interrelationship between mathematics and science. As one delves deeper into science, the existence of such a relationship becomes more apparent. A very important question on the minds of all science teachers is how to get students to understand and appreciate science as it becomes more mathematical in nature. Before going into the discussion of the pedagogical interrelationship between mathematics and science, it will help the reader to review a bit of the recent history of the development of science and mathematics programs.

In 1957, public interest in science and mathematics education reached its zenith when Sputnik blazed across the October sky. The results of this interest are legion today. Since that time we have seen special programs of science and mathematics developed at unbelievable rates. These programs, developed by scientists, mathematicians, and educators, all had one common antiseptic purpose. That is, to correct certain factors lethal to the school science and mathematics program. Paul Brandwein[1] has identified seven of these factors in the science program. They are as he sees them, first, the incredibly naive notion that scientists have developed or discovered a method that could solve all problems if properly applied, this method could be specified in steps collectively and called "problem solving" or the "scientific method." The steps are: (1) Define a problem; (2) Gather relevant data; (3) Form hypotheses; (4) Test the hypotheses; (5) Reach a conclusion. While no one probably would take issue with the five steps as a logical way of attacking a problem, it should be unmistakably clear to all, that there is no one firm method of the scientist. Second, technology, a product of science, was confused with the purposes, and the processes of science. The scientist and the technologist were equated. Few people realized that the scientist's major purpose is to understand heat, not how to make a heater. Few people realized that the scientist's search for truth is motivated by the Cardinal Newman principle that "knowledge is its own reward."

Third, the content of science was confused with a verified and certain body of facts. The real truth of the matter is that most of the things considered facts in 1850 in all probability, are not facts today. Very few people were aware of the failures of scientists.

Fourth, teaching science had become in effect, telling. Since science had given us the absolute truth, why go to the laboratory to verify

[1]Paul F. Brandwein, *The Revolution in Science Education: An Examination of the New Secondary Science Curriculums* (New York: Harcourt, Brace, and World, 1962), 5-10.

what was already known? For this kind of teaching, the lecture with an occasional demonstration sufficed. Problem-solving with its errors never dawned on anyone as being a vehicle for catching the essence of science.

Fifth, science was taught as history rather than discovery. Its facts were covered rather than uncovered. What little laboratory work that was in existence, was for problem-doing rather than problem-solving.

Sixth, the aim of a literacy in science could not be attained. Structure, in the "Jerome Bruner sense" was not taught or grasped.[2] The science courses did not provide the foundations in conceptual schemes that remain stable over a period of time. According to Bruner, "To learn structure, is to learn how things are related." To achieve literacy, the science teacher must teach basic and general ideas of science and then deepen and broaden knowledge in terms of these basic and general ideas.

Seventh, the creative and inquiring individual could not survive or develop in a fixed curriculum with its fixed methods and succession of unchallenged facts. Wherever provisions were made for the bright individuals, these too were fixed.

Eighth, I would like to add the confusion as to what science is. Many people confuse scientific models as reality, theories as the absolute truth. A fixed body of knowledge unchallenged and unreplenished eventually becomes superstitution. Science is more than knowledge; it is process working to keep itself dynamic and healthy. It corrects itself and it adds to itself. It is a process that refuses to let 1000 similar occurrences determine a law yet it will let one deviation from a law declare that law invalid. True science teaches to doubt. and in ignorance, to refrain.

The first step in developing a good science program was to correct the obvious defects described previously. Major curriculum revisions have been undertaken, largely with financial aid from the National Science Foundation. Several new courses have been developed. The Physical Science Study Committee, the Biological Sciences Curriculum Study, the Chemical Bond Approach Committee, and the Chemical Education Material Study were the first groups to develop new programs.[3]

In the meantime, new programs in secondary and elementary school mathematics were being developed. As a matter of historical fact, the mathematics groups began their work several years before the advent of Sputnik. The University of Illinois Committee on School Mathematics put its first textbook in use at the University of Illinois High School in September, 1952. Since then the UICSM program has been revised several times.

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[2]Jerome S. Bruner, *The Process Of Education* (Cambridge: Harvard University Press, 1963).

[3]Prince A. Jackson, Jr., *Science In The Schools* (Chestnut Hill: New England Catholic Education Center, Boston College, 1966), 12-16. In this monograph, all of the new science projects are described in great detail.

The School Mathematics Study Group (SMSG), representing the largest of the mathematics improvement programs, was formed in 1958. Funded by the National Science Foundation from its incipiency, this group has sought to improve mathematics from kindergarten through high school.

The University of Maryland Mathematics Project (UMMaP) took as its principal objective, to improve mathematics at the seventh and eighth grade level. The material developed by this group has been used in at least ten states.

The Boston College Mathematics Institute was organized by Reverend Stanley J. Bezuszka to develop materials for the last five years prior to college. The emphasis of the material is on the structure of mathematics.

The Ball State Teachers College Experimental Program was planned for grades seven through twelve. This program emphasizes mathematics through an axiomatic approach.

The Greater Cleveland Mathematics Program has developed and is now using improved mathematics materials in the lower grades of Cleveland, Ohio. The ultimate goal of GCMP is to develop improved materials for all grades in the Cleveland schools.

The University of Illinois Arithmetic Project was developed to give children a different view of mathematics. That is, to help them develop a fascination for work in mathematics. The project emphasizes "discovery" as its primary teaching tool.

The Stanford Project has as its prime objective the teaching of mathematics through the notions of sets and operations on sets. A second project, known as Mathematical Logic, also sponsored by Stanford University, emphasizes logic for gifted students of the fifth and sixth grades.

Although great attention, as evidenced by the developments of the above projects, has been given to the separate development of science and mathematics, there is virtually no evidence in the literature concerning the pedagogical interrelationship between mathematics and science. Although almost everyone agrees that the two areas are very closely related, no major project has been undertaken to explore this acknowledge relationship. Through the examples presented, it is hoped that this paper will attract the attention of science teachers and persuade them to improve science understandings through greater uses of mathematical ideas.

Perhaps the greatest difficulty encountered by the student in the science course is his inability to fully understand the use of models by science teachers to explain natural phenomena. Adding to this difficulty is the student's "way of thinking" about scientific theories. Most beginning students regard theories as "indisputable truths" rather than "approximations of the truth." Models are necessary because they allow us to represent reality and it is very important that students of science understand this realism. They must understand too, that models can not explain fully, natural phenomena because we do not know everything about the phenomena when we construct the model. In other words, the model is limited even from the outset because its creator has a limited knowledge of what the model supposedly represents.

When the student fully understands this, he understands immediately why it is necessary to revise the model and theory as we learn more about the phenomena supposedly explained by the model and the theory. One of the best written expositions on the uses and revisions of models is *The Restless Atom* by Alfred Romer. This book should be "must" reading for every beginning science student and layman.

How can we, teachers of science, do a better job of helping our students to understand the role of models in science? We can not ignore this question any longer because a basic understanding of models and their roles are absolutely necessary for the understanding of science today. The Bohr model of the atom assists us in answering many questions in the physical sciences as well as in the prediction of the behavior of matter under specified conditions. The kinetic theory of heat is another excellent example of how models greatly assist in the explanation of natural phenomena. However, the limitations of models in the explanation of natural phenomena must be understood by science students. The wave and quantum theories of light present us with an example of this limitation. Both models are excellent, but neither can explain light phenomena adequately.[4] The use of models in explaining nuclear transmutations of elements is another example of the limitation of models.

In mathematics, teachers have been able to overcome the "model" problem to a much greater extent than teachers of science. In mathematics, "approximation" is the equivalence of "model" in science. How is this done in mathematics?

The mathematician deals with an abstraction that we call "number." He represents this abstraction with a symbol that we call "numeral." We use under ordinary conditions a system of numeration with a "base ten" where we understand that a system of numeration is a method of writing the names of the members of an infinite sequence of numbers. In the "base ten" system we use the symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These symbols are adequate as long as we are expressing rational numbers.[5] The use of "approximations" becomes necessary when we have to express irrational numbers.[6] The numeral expressing the exact value of the square root of 2 is $\sqrt{2}$. But students seemingly get a better idea of the place of the number

[4]The writer recalls vividly his classes in Physics at Harvard University where the students would jokingly tell each other in the class to use the wave model on Monday, Wednesday, and Friday; the quantum model on Tuesday, Thursday, and Saturday; and pray for guidance on Sunday.

[5]If a and b are any two integers with $b \neq O$, then a/b is called a rational number. Any real number that can be expressed in this form is called a rational number.

[6]A real number is called an irrational number if it is not a rational number. That is, a real number that can not be expressed as the ratio of two integers, a/b where $b \neq O$.

of the real number line if it can be expressed in decimal or fractional form. So mathematics teachers write 1.41 as an approximation. So, as in science, this "approximation" or "model" will suffice under ordinary conditions. When a better "approximation" is needed, 1.414, or 1.4142, 1.41421, 1.414214, . . . may be used depending upon the degree of accuracy needed. The transcendental irrational number "pi", the ratio of the circumference of a circle to its diameter, is another example through which the concept of "approximation" may be strengthened. Some mathematics teachers use 22/7 as an "approximation".[7] Another popular "approximation" is 3.1416. A better "approximation" is 3.14159.

Since it is possible for the student of mathematics to construct these "approximations" himself, the concept of the use of "approximation" in mathematics is easier to comprehend than the concept of the use of "model" in science. However, there is a pedagogical interrelationship between "approximation" as it is used in mathematics and "model" as it is used in science. Teachers of science could do much to clarify the obscurities that surround the use of "model" in science. Since models are approximations of what we consider to be reality, why not use mathematical approximations to show the role of models in science. This could, and probably would demonstrate to science students why we do not consider a "model" as being wrong whenever it is replaced by a better "model". Another very important point that could be clarified is the distinction that scientists make between what is wrong and what is highly inaccurate.

Most secondary schools today offer programs evolving from the various new curricula in science and in mathematics referred to at the beginning of this paper. Most college mathematics departments teach freshman mathematics from a "set-theory" standpoint. Yet, most secondary school science students and college science students hardly, if ever, utilize the concept of "sets" in their science work. The concept of "sets" can be of great value in the science laboratory. There is a very strong pedagogical interrelationship between mathematics and science in the area of "sets". An exploration of this relationship implies that it can be used advantageously in the science class. As we think about the concept of a set in a science setting, we find that it is inherently a science concept. A set is thought of as a collection of objects with each object having the distinguishing characteristic of the set.[8] The latter part of the above statement allows us to decide whether a given object belongs or does not belong to a particular set. The distinguishing characteristic of a set may take shape in many forms. We may think of a set R of real numbers defined by $\{ X \mid X \varepsilon R \text{ and } O < X < 1 \}$. All members of this in-

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[7]The writer considers this to be the worst of the approximations because 22/7 in addition to being a poor approximation is a ratio of two integers. This sometimes conveys to the student that "pi" is a rational number even more so than the highly used 3.14 which is also a rational number.

[8]Admittedly this definition is formulated to appeal to the intuition. For a rigorous discussion on the notion of a set, see Susanne K. Langer, *An Introduction to Symbolic Logic* (New York: Dover Publications, Inc., 1953), 115-16.

finite set share the property or have the characteristic that they are to the right of 0 and to the left of 1 on the real number line. Sets of this type are encountered frequently in mathematics. They are especially important because they allow mathematicians to look at many objects at the same time by putting objects with common properties in the same class.[9] Thus, consideration of a class allows consideration of infinitely many things simultaneously. A universe U is considered to be the set of all objects under consideration. It is from the universe U where we extract the sets being considered.

In science we might consider the set of all ratio-active elements, the set of solid substances having specific heats greater than the specific heat of water, and the set of all solid substances having densities less than the density of water (at 760mm of pressure and 4°C).[10] The universes from which the above sets have been drawn are the set of all chemical elements, the set of all solid substances having finite specific heats, and the set of all solid substances having finite densities. In each of these universes, there are other sets. In the universe of solid substances having finite specific heats, there is a set of these substances with specific heats less than that of water (the specific heat of water is 1 cal/gmC°), there is the set of substances with specific heats equal to the specific heat of water, and there is the set of substances having specific heats greater than that of water. In the same manner, the universe of solid substances having finite densities can be described. As is true of all sets, each subset of the universes described above has a distinguishing characteristic. The elements of each subset can be distinguished by the distinguishing characteristic of the subset.

The algebra of sets can be applied to sets of science objects. The intersection of the set of solids having densities greater than the density of water and the set of solids having specific heats less than the specific heat of water is not an empty set. Aluminum has a density of 2.7 gm/cm^3 and a specific heat of 0.22 cal/gmC°. Thus at least one substance is in both sets. There are also substances like iron, lead, and copper that are in one of the sets (having specific heats less than the specific heat of water) but not in the other (having densities greater than the density of water).[11]

In mathematics, the idea of a set opens the door to relations and functions. A relation is a set whose elements are ordered pairs. An ordered pair is a pair of objects that occur in a specified order.[12] Since each element of a relation is an ordered pair, we may form a new set consisting only of the first elements of the ordered pairs making up the relation. This set is called the domain of the relation.

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[9]Robert R. Stoll, *Sets, Logic, and Axiomatic Theories* (San Francisco: W. H. Freeman and Company, 1961), 13.
This is the essential point of Georg Cantor (1845-1918), the German mathematician, who is considered as the originator of set theory.

[10]It can be shown experimentally that water under 1 atmosphere of pressure has its greatest density, 1 gram per cubic centimeter, at 4° Centigrade.

[11]The specific heats of iron, lead, and copper are respectively 0.11, 0.031 and 0.093 cal/gmC°. Their respective densities are 7.9, 11.3, and 8.9 gm/cm^3.

[12]More precisely, the ordered pair $(a,b) = \{ (a), (a,b) \}$.

We may form a second set consisting only of the second elements of the ordered pairs making up the relation. This set is called the range of the relation. A relation having the property that no two distinct ordered pairs belonging to the set have the same first element is called a function. Another way of looking at the function is to consider it as a law of correspondence that associates with every member of a set A, a unique element of a set B.

In science, the search for truth is essentially a search for order. Scientists are constantly looking for relationships. When relationships are found then the next step is to express them as functions. When this is done, the correspondence may be called a law or principle. The difficulty of finding exact relations can best be demonstrated by counting the number of laws and principles known to science today.

In the laboratory, data are collected and graphs[13] are constructed to find if any relationships exist. However, it is easy to see that physical conditions limit the domain of any existing relation. If one wanted to show the relationship between heat and temperature it is obvious that the amount of heat used is limited by the limitations of the laboratory.

An easy experiment may be devised by the student in the following way. Obtain a container of water equipped with a Centigrade thermometer and apparatus for measuring the amount of heat supplied to the water. As calories of heat are supplied to the water, it is noticed that the temperature of the water rises. In this case the two variables are the heat being supplied and the resulting temperatures. Taking the heat being supplied as the independent variable and the resulting temperatures as the dependent variable, the student could graph the relation from the set of ordered pairs consisting of heat and temperature. In this case, the domain consists of calories of heat and the range of the relation consists of the resulting temperatures. The resulting graph of the relation is a straight line as revealed in Figure 1.

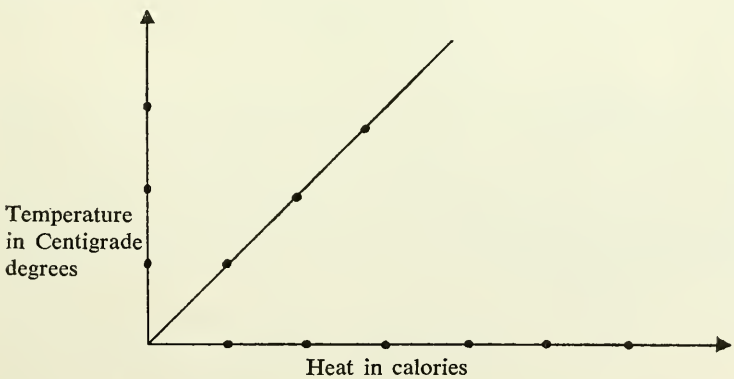

Figure 1. Relationship of Heat and Temperature

[13]The set of points corresponding to the ordered pairs belonging to a relation is called the graph of the relation.

Since the graph is a straight line, the student will conclude immediately that the relation is a function.

To discover more about the inclination of the line, the student can extend his experiment to include two other liquids in the same quantity as was the water. Repeating the experiment two more times using the same amount of heat and starting from the same temperature, he obtains the graphs of two different straight lines as can be seen in Figure 2.

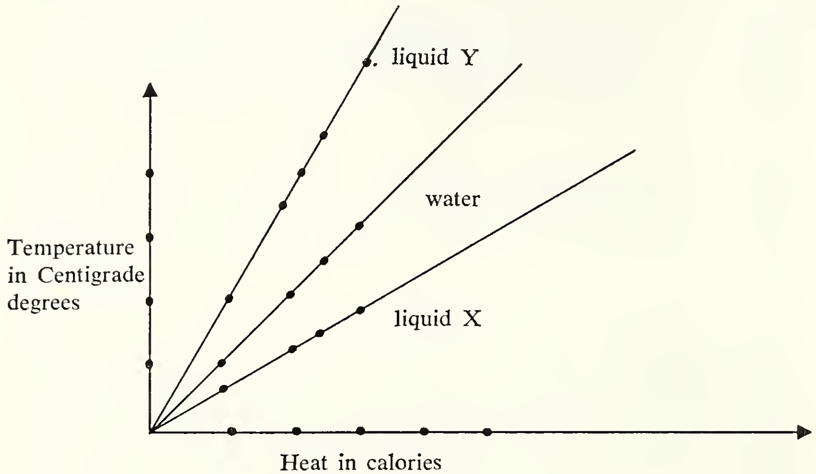

Figure 2. Relationship of Heat and Temperature of Several Liquids

Upon investigation, the student may reach the conclusion that the inclination of the line is related to the specific heat of the liquid. He may also want to investigate the relationship with densities of the liquids used. The mathematical concept of slope[14] may be brought into the experiment. In such an experiment, the student has many possibilities to investigate and should be encouraged to do so.

As an example of a non-linear relation, the science student should be encouraged to study Boyle's Law—the relation between pressure of a gas and volume of a gas when the temperature of the gas is kept constant. In the school laboratory, the student can collect the data based on several changes in volume. Using the volume recordings as the domain and the resulting temperatures as the range, the graph can be constructed from the set of ordered pairs. These facts are shown in Figure 3.

[14]The tangent of the angle of inclination.

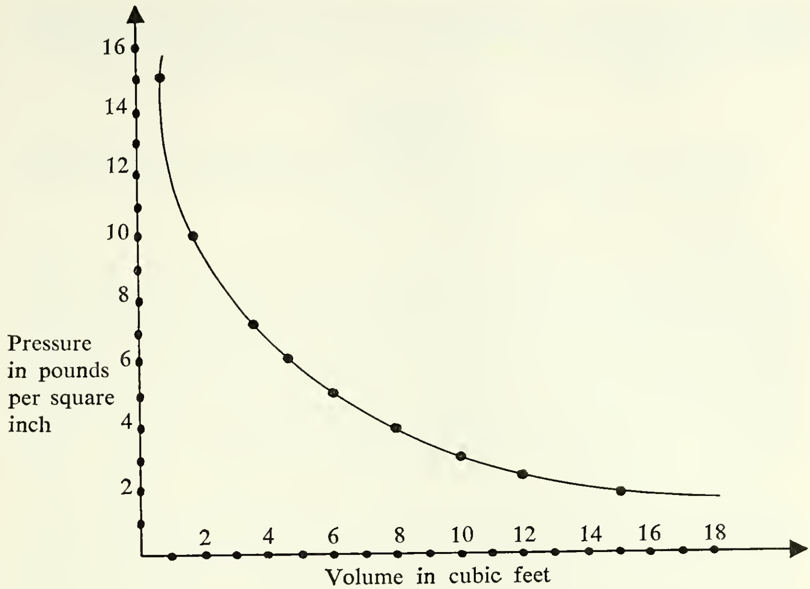

Figure 3. Relationship of Volume to Pressure

From the graph, the student concludes immediately that this is an inverse relation. If his observations in the laboratory are good, he can derive Boyle's law by studying the graph carefully. That is, he would discover, $PV=K$.

Upon further investigation of the graph, he finds that the graph is a hyperbola and can easily prove that it is a function.[15]

The pedagogical interrelationships between mathematics and science discussed in this paper are not designed to add to the vast repertoire of mathematics and science. However, it is hoped that greater exploration of them will assist the science students, especially the secondary and beginning college students, to gain a keener insight into the very close relationship existing between mathematics and science. The science teacher can no longer legitimately complain about meager mathematical backgrounds of science students as in former years. The students entering the secondary school science classes and beginning college science classes are in command of rich mathematical experiences, both in techniques and in ideas. If science teachers encourage these students to utilize these mathematical ideas in doing science, the outcomes, in terms of student growth and development in science, could reach undreamed of heights. In utilizing these mathematical concepts in doing science, the students will enjoy

[15]Some teachers have their students to check the graph to see whether any line drawn parallel to the ordinate will intersect the graph in more than one point. If yes, then the relation is not a function. If no, then the relation is a function.

science rather than look upon it as just another obstacle to overcome before graduation.

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