

**FACULTY RESEARCH EDITION**  
of  
**The Savannah State College Bulletin**

*Published by*

**The Savannah State College**

Volume 20, No. 2

Savannah, Georgia

December, 1966

HOWARD JORDAN, JR., *President*

**Editorial Committee**

Blanton E. Black

J. Randolph Fisher

Mildred W. Glover

Joan L. Gordon

Elonnie J. Josey

Charles Pratt

Forrest O. Wiggins

JOHN L. WILSON, *Chairman*

*Articles are presented on the authority of their writers, and neither the Editorial Committee nor Savannah State College assumes responsibility for the views expressed by contributors.*

BOUND BY THE NATIONAL LIBRARY BINDERY CO. OF GA.

75036

## Table of Contents

|                                                                                                           |    |
|-----------------------------------------------------------------------------------------------------------|----|
| A Guide to the Study of Current Introduction to<br>Education Textbooks                                    | 7  |
| Charles I. Brown .....                                                                                    |    |
| Synthesis of 4:6 Thio 1, 3, 5-triazine Derivatives II                                                     | 10 |
| Kamalaker B. Raut .....                                                                                   |    |
| The Humanities                                                                                            | 12 |
| James H. Hiner .....                                                                                      |    |
| The Influence of Religion on the Political Process in<br>Burma                                            | 22 |
| Johnny Campbell .....                                                                                     |    |
| Creating a National Sense of Direction in Industrial Arts                                                 | 34 |
| Richard Cogor .....                                                                                       |    |
| The Teaching of Mathematical Induction                                                                    | 36 |
| William M. Perel .....                                                                                    |    |
| The Evolutionary Role of the International Labor Organization                                             | 40 |
| Sarvan K. Bhatia .....                                                                                    |    |
| What Motivates Students in the Choice of Subject Majors                                                   | 48 |
| Dorothy C. Hamilton .....                                                                                 |    |
| A Device for the Improvement of Study Habits                                                              | 55 |
| Maurice A. Stokes .....                                                                                   |    |
| On Variation of Velocity and Pressure Behind and Along a<br>Shock Surface in Lagrangian Coordinate System | 65 |
| Nazir A. Warsi .....                                                                                      |    |
| On Vorticity Behind a Shock Surface in Lagrangian Coordinate<br>System                                    | 68 |
| Nazir A. Warsi .....                                                                                      |    |
| On Gradients of Specific Volume and Pressure Behind a Shock<br>Surface in Lagrangian Coordinate System    | 71 |
| Nazir A. Warsi .....                                                                                      |    |
| The Community College: An American Innovation                                                             | 73 |
| Philip D. Vairo .....                                                                                     |    |
| Force Field Calculations in Octahedral Water Complexes                                                    | 76 |
| Venkataraman Ananthanarayanan .....                                                                       |    |
| The Negro in International Affairs-Prospects for the Future                                               | 80 |
| George L-P Weaver .....                                                                                   |    |
| The Law of Karma as Reflected in Hinduism, Buddhism<br>and Jainism                                        | 85 |
| Samuel Williams .....                                                                                     |    |
| Economic Growth and Income Distribution                                                                   | 92 |
| Sarvan K. Bhatia .....                                                                                    |    |

## Table of Contents – (Continued)

|                                                                                                                                                                             |     |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| India's Experience in Developmental Planning<br>Kanwal Kumar .....                                                                                                          | 98  |
| Utilizing Emerging New Instructional Materials and Mechanical<br>Devices-Implications for the Library<br>Dorothy B. Jamerson .....                                          | 103 |
| The Moynihan Report: A Critical Analysis<br>Isaiah McIver .....                                                                                                             | 108 |
| Modern Art: The Celebration of Man's Freedom<br>Phillip J. Hampton .....                                                                                                    | 122 |
| A Review of "The Use of Selected Technical Language<br>as a Means of Discovering Elementary Teachers'<br>Operational Definitions of Teaching"<br>Thelma Moore Harmond ..... | 128 |
| An Analysis of NTE Scores and Quality Point Ratios of<br>Selected SSC Graduates from 1961 through 1966<br>Martha W. Wilson .....                                            | 141 |
| The Man Behind "Trees"<br>James A. Eaton .....                                                                                                                              | 147 |
| Watts: A Tragedy of Errors<br>Elonnie J. Josey .....                                                                                                                        | 153 |
| Personal Characteristics in Secondary School Social Studies<br>Student Teachers as Related to Certain Measures of<br>Potential Teacher Behavior<br>Shia-ling Liu .....      | 159 |
| Some Possible Ways of Improving Instruction in Our Colleges<br>Robert D. Reid .....                                                                                         | 165 |
| Needed: A Program to Save Freshmen!<br>James A. Eaton .....                                                                                                                 | 174 |

# On Variation of Velocity and Pressure Behind and Along A Shock Surface In Lagrangian Coordinate System

by

Nazir A. Warsi

## 1. INTRODUCTION

The flow of a perfect gas in Lagrangian coordinate system is given by [1]

$$(1.1) \quad U_{\alpha/}^i \tau_{\alpha/} \rho_{\alpha/} - \tau_{\alpha/}^i U_{\alpha/}^i = 0$$

$$(1.2) \quad U_{\alpha/}^i U_{\alpha/}^j \rho_{\alpha/} + \tau_{\alpha/} \rho_{\alpha/} = 0$$

$$(1.3) \quad p_{\alpha/} = \exp\left\{\eta_{\alpha/} / (C_U)\right\} p^r$$

$$(1.4)a \quad \partial \eta_{\alpha/} / \partial t = 0$$

$$(1.4)b \quad \eta_{\alpha/} U_{\alpha/}^i = 0$$

where  $f_{\alpha/}$  is a quantity in the region  $\alpha/$  and  $2/,1/$  denote the regions behind and in front of the shock surface.

Also, the conservation equations across the shock surface are given by [2]

$$(1.5) \quad [U^i] = -h_{n/} \delta z/ \tau_{1/} X^i$$

$$(1.6) \quad [p] = -h_{n/} \delta z/ \tau_{1/}$$

$$(1.7) \quad [\tau] = -\frac{\delta z/}{h_{n/}} U_{1n/}$$

where

$$(1.8) \quad \delta z/ = \frac{2}{\gamma+1} \cdot \frac{C_{1/}^2 - h_{n/}^2 \tau_{1/}}{h_{n/}^2 \tau_{1/}}$$

## 2. VARIATIONS

Differentiating (1.3) and applying (1.4)b, we get

$$(2.1)a \quad U_{\alpha/}^i p_{\alpha/} \rho_{\alpha/} = C_{\alpha/}^2 \rho_{\alpha/} U_{\alpha/}^i$$

or

$$(2.2)b \quad U_{\alpha/}^i p_{\alpha/} \rho_{\alpha/} = -\frac{C_{\alpha/}^2}{\tau_{\alpha/}^2} \tau_{\alpha/} \rho_{\alpha/} U_{\alpha/}^i$$

In consequence of (1.1) and (2.1), we get

$$(2.3) \quad U_{\alpha/}^i U_{\alpha/}^j U_{\alpha/}^k = \frac{C_{\alpha/}^2}{3} U_{\alpha/}^i U_{\alpha/}^j$$

Multiplying (1.2) by  $U_{\alpha/}^j$  and using (2.3), we get

$$(2.4) \quad \tau_{\alpha/} U_{\alpha/}^j p_{\alpha/} \rho_{\alpha/} = -\frac{C_{\alpha/}^2}{3} U_{\alpha/}^i U_{\alpha/}^j$$

Now, let us define quantities  $L_j^i$  such that

$$(2.5) \quad L_\alpha^i \stackrel{\text{def}}{=} x_{i,\alpha}$$

and

$$(2.6) \quad L_3^i \stackrel{\text{def}}{=} (u_{1j}^i - h_{n_j} \tau_{1j} \delta_{z_j} X^i)$$

If  $M_j^i$  are quantities such that  $|L_j^i| \neq 0$ , and

$$(2.7) \quad M_\kappa^i L_j^\kappa = \delta_j^i$$

$$(2.8) \quad M_i^\kappa M_\kappa^j = \delta_i^j,$$

then  $M_j^i$  are the inverse of  $L_j^i$ . Hence, we have

$$(2.9) \quad M_j^i = \frac{\text{cofactor of } L_j^i \text{ in } |L_j^i|}{|L_j^i|}.$$

Now, from (2.5) and (2.6), we get

$$(2.10) \quad |L_j^i| = \begin{vmatrix} L_1^1 & L_2^1 & L_3^1 \\ L_1^2 & L_2^2 & L_3^2 \\ L_1^3 & L_2^3 & L_3^3 \end{vmatrix}$$

The values of  $L_1^1$ ,  $L_2^1$ , etc. can be substituted from (2.5) and (2.6) and  $|L_j^i|$  can be calculated. Let  $L = |L_j^i|$ .

We have the following theorems.

**THEOREM 2. 1:** The values of  $M_i^\alpha$  are given by

$$(2.11) \text{ a } M_i^\alpha = \frac{1}{L} \epsilon^{\alpha\beta} \epsilon_{ijk} x_{j,\beta} (u_{1j}^k - h_{n_j} \tau_{1j} \delta_{z_j} X^k)$$

or

$$(2.11) \text{ b } M_i^\alpha = \frac{1}{L} \epsilon^{\alpha\beta} \epsilon_{ijk} (u_{1j}^k + u_{1n_j} \delta_{z_j} X^k)$$

Where

$\epsilon^{\alpha\beta}$  are quantities, having values

(i) +1 when  $\alpha = 1, \beta = 2$

(ii) -1 when  $\alpha = 2, \beta = 1$

(iii) 0 when  $\alpha = \beta$

and  $\epsilon_{ijk}$  are the components of an isotropic tensor having numerical values

(i) +1 when  $ijk$  is an even permutation of 123

(ii) -1 when  $ijk$  is an odd permutation of 123

(iii) 0 when  $ijk$  are the same.

**PROOF:** From (2.9), we get

$$(2.12) \quad M_i^\alpha = \frac{\text{cofactor of } L_\alpha^i \text{ in } L}{L}$$

In consequence of (2.5), (2.6) and (2.10) we get (2.11)a which by virtue of the relation  $-\dot{h}_{n_j} \tau_{2j} = V_{2n_j}^i$ , gives (2.11)b.

**THEOREM 2.2:** Quantities  $M_i^3$  are given by

$$(2.13) M_i^3 = \frac{1}{2L} \epsilon^{\alpha\beta} e_{\alpha\beta} X_i$$

where  $e_{\alpha\beta}$  are the components of a surface tensor skew-symmetric in  $\alpha, \beta$  and defined by

$$(2.14) e_{\alpha\beta} \stackrel{\text{def}}{=} \epsilon_{ijk} X^i X_{j,\alpha} X_{k,\beta}$$

**PROOF:** From (2.9) we have

$$(2.15) M_i^3 = \frac{\text{Cofactor of } L_{2i}^i \text{ in } L}{L}$$

which with the help of (2.5), (2.6) and (2.10) gives (2.13).

**THEOREM 2.3:** The variation of velocity along the surface behind the shock is given by

$$(2.16) U_{2/j}^i X_{j,\alpha}^i = U_{2/j}^i X_{j,\alpha}^i + \dot{h}_{n_j} \tau_{2j} \delta_{2j} e_{\alpha\beta} g^{\beta\gamma} X_{j,\alpha}^i + \frac{4C_{2j} C_{2j,\alpha} X^i}{(\gamma+1)\dot{h}_{n_j} \tau_{2j}} + \left\{ 4 + \delta_{2j} (\gamma+1) \dot{h}_{n_j}^2 \tau_{2j}^2 - 4C_{2j}^2 + \dot{h}_{n_j} \tau_{2j} \right\} \frac{(\dot{h}_{n_j} \tau_{2j})_{,\alpha}}{(\gamma+1)\dot{h}_{n_j}^2 \tau_{2j}^2}$$

where  $g^{\alpha\beta}$  is a reciprocal tensor of the first fundamental tensor  $g_{\alpha\beta}$  of the surface and  $e_{\alpha\beta}$  the second fundamental tensor of the surface.

**PROOF:** Differentiating (1.5) and (1.8) partially with respect to the surface parameters  $y^\alpha$ , we get

$$(2.17) U_{2/j}^i X_{j,\alpha}^i = U_{2/j}^i X_{j,\alpha}^i - \dot{h}_{n_j} \tau_{2j} \delta_{2j,\alpha} X^i - \dot{h}_{n_j} \delta_{2j} \tau_{2j,\alpha} X^i - \dot{h}_{n_j,\alpha} \delta_{2j} \tau_{2j} X^i - \dot{h}_{n_j} \tau_{2j} \delta_{2j,\alpha} X^i$$

and

$$(2.18) \delta_{2j,\alpha} = \frac{4}{(\gamma+1)\dot{h}_{n_j}^2 \tau_{2j}^3} \left\{ \dot{h}_{n_j} \tau_{2j} C_{2j} C_{2j,\alpha} + (\dot{h}_{n_j} \tau_{2j})_{,\alpha} (\dot{h}_{n_j} \tau_{2j}^2 - C_{2j}^2 - \tau_{2j} \dot{h}_{n_j}) \right\}$$

By making use of Weingarten equation [3] in (2.17) and applying (2.18), we get (2.16).

**THEOREM 2.4:** The variation of pressure along the surface behind the shock, is given

$$(2.19) p_{2/j}^i X_{j,\alpha}^i = p_{2/j}^i X_{j,\alpha}^i - (\dot{h}_{n_j}^2 \tau_{2j})_{,\alpha} \delta_{2j} - \dot{h}_{n_j}^2 \tau_{2j} \delta_{2j,\alpha}$$

**PROOF.** Differentiating (1.6) partially with respect to  $y^\alpha$ , we get (2.19).

## REFERENCES

1. Warsi, N. A. (1964) On Geometry of Gas Flows in Lagrangian Coordinate System. Savannah State College Reserach Bulletin, Vol. 18, No. 2.
2. Warsi, N. A. (1965) Flow Parameters Behind Three Dimensional Shock Wave. Savannah State College Faculty Research Bulletin, Vol. 19, No. 2.
3. Eisenhart, L. P. (1941). Introduction to Differential Geometry. Princeton University Press.

# On Vorticity Behind A Shock Surface In Lagrangian Coordinate System

by

Nazir A. Warsi

## 1. INTRODUCTION

The author [1] has discussed the variation of velocity and pressure along and behind the shock surface. The results derived by him will be used in this paper in finding the vorticity behind the shock wave.

## 2. DERIVATIVE OF VELOCITY

If  $S_{ij}$  are the quantities defined by

$$(2.1) S_{ij} \stackrel{\text{def}}{=} U_{2/\alpha}^{\alpha} L_i^{\alpha} L_j^{\alpha},$$

then we have the following theorems.

Theorem 2.1: Quantities  $S_{\alpha\beta}$  are given by

$$(2.2) S_{\alpha\beta} = U_{2/\alpha}^{\alpha} X_{,\alpha}^{\alpha} X_{,\beta}^{\alpha} + \dot{h}_{n/\tau_1} \delta_{\tau_1} d_{\alpha\beta}$$

Proof: Putting  $i = \alpha, j = \beta$  in (2.1), we get

$$(2.3) S_{\alpha\beta} = U_{2/\alpha}^{\alpha} L_{\alpha}^{\alpha} L_{\beta}^{\alpha}$$

which, in consequence of (2.5) of [1] gives

$$(2.4) S_{\alpha\beta} = U_{2/\alpha}^{\alpha} X_{,\alpha}^{\alpha} X_{,\beta}^{\alpha}$$

On substituting from (2.16) of [1] and using  $X^i X_{,\alpha}^i = 0$ ,

$$(2.4) \text{ gives } (2.2).$$

Theorem 2.2: Quantities  $S_{\alpha 3}$  are given by

$$(2.5) S_{\alpha 3} = \tau_1 (1 + \delta_{\tau_1}) \{ p_{1/\alpha} - (\dot{h}_{n/\tau_1}^2 \tau_1)_{,\alpha} \delta_{\tau_1} - \dot{h}_{n/\tau_1}^2 \tau_1$$

Proof: Putting  $i = \alpha, j = 3$  in (2.1), we get

$$(2.6) S_{\alpha 3} = U_{2/\alpha}^{\alpha} L_{\alpha}^{\alpha} L_3^{\alpha}$$

which, with the help of (2.5) and (2.6) of [1] gives

$$(2.7) S_{\alpha 3} = U_{2/\alpha}^{\alpha} X_{,\alpha}^{\alpha} (U_{1/\alpha}^{\alpha} - \dot{h}_{n/\tau_1} \delta_{\tau_1} X^{\alpha}).$$

We get (2.5) if we substitute from (2.16) of [1] in (2.7).

Theorem 2.3: The quantities  $S_{3\alpha}$  are given by

$$(2.8) S_{3\alpha} = (U_{1/\alpha}^{\alpha} - \dot{h}_{n/\tau_1} \delta_{\tau_1} X^{\alpha}) \{ U_{3/\alpha}^{\alpha} X_{,\alpha}^{\alpha} + \dot{h}_{n/\tau_1} \delta_{\tau_1} d_{\alpha\beta} g^{\beta\gamma} X_{,\gamma}^{\alpha} \\ + \frac{4C_{1/\alpha} C_{3/\alpha} X^{\alpha}}{(\gamma+1)\dot{h}_{n/\tau_1}} + \frac{(\dot{h}_{n/\tau_1} \tau_1)_{,\alpha}}{(\gamma+1)\dot{h}_{n/\tau_1} \tau_1} \cdot \left[ \{ 4 + \delta_{\tau_1} (\gamma+1) \dot{h}_{n/\tau_1}^2 \tau_1^2 - 4C_2^2 + \dot{h}_{n/\tau_1} \tau_1 \} \right]$$

Proof: Putting  $i = 3, j = \alpha$  in (2.1), we get

$$(2.9) \mathcal{S}_{3\alpha} = U_{2/3, \alpha}^{\alpha} L_3^{\alpha} L_{\alpha}^{\alpha},$$

which, by virtue of (2.5) and (2.6) of [1], gives

$$(2.10) \mathcal{S}_{3\alpha} = U_{2/3, \alpha}^{\alpha} X_{3, \alpha}^{\alpha} (U_{2/3, \alpha}^{\alpha} - \dot{h}_{n/3} \tau_{2/3} \delta_{\tau_j} X^{\alpha}).$$

using (2.17) of [1], we get (2.8).

Theorem 2.4: The quantity  $\mathcal{S}_{33}$  is given by

$$(2.11) \mathcal{S}_{33} = \left\{ C_{2/3}^2 - \frac{\gamma-1}{2} \delta_{\tau_j} (\delta_{\tau_j} + 2) \dot{h}_{n/3}^2 \tau_{2/3}^2 \right\} \frac{U_{2/3, i}^i}{3}$$

Proof: Putting  $i = 3, j = 3$  in (2.1), we get

$$(2.12) \mathcal{S}_{33} = U_{2/3, 3}^3 L_3^3 L_3^3$$

which, in consequence of (2.6) of [1], gives

$$(2.13) \mathcal{S}_{33} = U_{2/3, 3}^3 (U_{2/3, 3}^3 - \dot{h}_{n/3} \tau_{2/3} \delta_{\tau_j} X^3) (U_{2/3, 3}^3 - \dot{h}_{n/3} \tau_{2/3} \delta_{\tau_j} X^3)$$

On substitution from (2.17) of [1], (2.13) gives (2.11).

Theorem 2.5: The value of  $U_{2/3, i}^i$  is given by

$$(2.14) U_{2/3, i}^i = \frac{3L^2 \{ S_{\alpha\beta} M_{\alpha}^{\beta} \cdot M_{\alpha}^{\beta} + 2 S_{(\alpha\beta)} M_{\alpha}^{\beta} M_{\alpha}^{\beta} \}}{3L^2 - 4e^{\frac{\gamma}{2}} \{ C_{2/3}^2 - \frac{\gamma-1}{2} \delta_{\tau_j} (\delta_{\tau_j} + 2) \dot{h}_{n/3}^2 \tau_{2/3}^2 \}}$$

Proof: From (2.1), (2.7) and (2.8), we get

$$(2.15) U_{2/3, \delta}^{\delta} = S_{\delta j} M_{\delta}^j M_{\delta}^j$$

which, on putting  $s=r$ , yields

$$(2.16)a \quad U_{2/3, \kappa}^{\kappa} = S_{\delta j} M_{\delta}^j M_{\delta}^j$$

or

$$(2.16)b \quad U_{2/3, \kappa}^{\kappa} = S_{\alpha\beta} M_{\delta}^{\alpha} M_{\delta}^{\beta} + 2 S_{(\alpha\beta)} M_{\delta}^{\alpha} M_{\delta}^{\beta} + S_{33} M_{\delta}^3 M_{\delta}^3.$$

By virtue of (2.13) of [1], (2.12) and (2.16)b, we get (2.14).

Theorem 2.6: The quantity  $\mathcal{S}_{33}$  is completely determined in terms of the quantities of region, in front of the shock surface, with the help of (2.17)

$$\mathcal{S}_{33} = \frac{3L^2 \{ C_{2/3}^2 - \frac{\gamma-1}{2} \delta_{\tau_j} (\delta_{\tau_j} + 2) \dot{h}_{n/3}^2 \tau_{2/3}^2 \} \{ S_{\alpha\beta} M_{\delta}^{\alpha} M_{\delta}^{\beta} + 2 S_{(\alpha\beta)} M_{\delta}^{\alpha} M_{\delta}^{\beta} \}}{3L^2 - 4e^{\frac{\gamma}{2}} \{ C_{2/3}^2 - \frac{\gamma-1}{2} \delta_{\tau_j} (\delta_{\tau_j} + 2) \dot{h}_{n/3}^2 \tau_{2/3}^2 \}}$$

Proof: Applying (2.14) to (2.11), we get (2.17).

Theorem 2.7: The components of vorticity vector  $\omega_{2/3}^k$  behind the shock surface are given by

$$(2.18) \omega_{2/3}^k \in \delta^{ijk} S_{lm} M_{\delta}^l M_{\delta}^m$$

Proof: From all the discussions of article 2, it is obvious that

$M_j$ ,  $S_{\delta j}$  are completely determined in terms of the quantities of the region in front of the shock surface. Hence from

(2.1) and (2.7),

(2.8) of [1], we get

$$(2.19) U_{2/3, i}^i = S_{lm} M_{\delta}^l M_{\delta}^m$$



The vorticity vector  $\omega_{\alpha j}$  is given by

$$(2.20) \quad \omega_{\alpha j} = \epsilon^{ijk} u_{\alpha j, k}$$

Substituting from (2.19) for  $\omega_{\alpha j} = 2$  in (2.20), we get (2.19).

#### REFERENCES

1. Warsi, N. A. (underpublication): On Variation of Velocity and Pressure Behind and Along Shock Surface. Savannah State College Faculty Research Bulletin.
2. Friedrichs-Courants (1948): Supersonic Flow and Shock Waves. Interscience Publishers, New York.
3. Howarth, L. (1953): Modern Developments in Fluid Dynamics. Vols. 1, 2. Oxford University Press.

# On Gradients of Specific Volume and Pressure Behind A Shock Surface In Lagrangian Coordinate System

by

Nazir A. Warsi

## 1. INTRODUCTION

The author [1, 2] has discussed the variation of flow parameters and vorticity behind a shock surface in Lagrangian Coordinate System. This article deals with the gradients of pressure and density using the results obtained in [1, 2].

## 2. GRADIENT OF DENSITY

The specific volume shock strength is given by [3]

$$(2.1) \delta z_j = [z] / z_{1j}$$

Differentiating (2.1) partially with respect to  $x^{\alpha}$ , we get

$$(2.2) z_{2/j} x^j_{,\alpha} = z_{1/j,\alpha} (1 + \delta z_j) + z_{1j} \delta z_{j,\alpha}$$

Also, we have [4, 3]

$$(2.3) u_{2j} \dot{z}_{2/j} - \frac{z_{2j}}{z} u_{2/j} \dot{z}_{2j} = 0$$

$$(2.4) [u^i] = -h \eta / \delta z_j z_{1j} x^i$$

On substitution from (2.3), (2.1) and (2.4), the equation (2.2)

becomes (2.5)  $z_{2/j} (u_{1j} - h \eta / z_{1j} \delta z_j x^j) = z_{1j} (1 + \delta z_j) u_{2/j}$

which, in consequence of (2.14) of [2], becomes

$$(2.6) z_{2/j} (u_{1j} - h \eta / z_{1j} \delta z_j x^j) = \frac{z_{1j} (1 + \delta z_j) 3L^2 \left\{ \frac{\delta_{\alpha\beta} M_{\alpha}^{\alpha} M_{\beta}^{\beta} + 2\delta_{(\alpha\beta)} M_{\alpha}^{\alpha} M_{\beta}^{\beta} \right\}}{3L^2 - 4e_{12}^2} \left\{ \frac{c_{1j}^2}{2} - \frac{\gamma-1}{2} \delta z_j (\delta z_j + 2) h^2 \eta / z_{1j}^2 \right\}}$$

Now, let us define quantities  $Z_i$  such that

$$(2.8) Z_i = z_{2/j} L_i^j$$

Hence, we have the following theorem.

**Theorem 2.1:** *The quantities  $Z_{\alpha}$  are given by the equation*

$$(2.9) Z_{\alpha} = z_{1/\alpha} (1 + \delta z_j) + z_{1j} \delta z_{j,\alpha}$$

**Proof:** Putting  $i = \alpha$  in (2.8), we get

$$(2.10) Z_{\alpha} = z_{2/j} L_{\alpha}^j$$

which, with the help of (2.5) of [1], becomes

$$(2.11) Z_{\alpha} = z_{2/j} x^j_{,\alpha}$$

Equation (2.9) is readily obtained if we substitute from (2.6) in (2.11).

Theorem: 2.2: The quantity  $\mathcal{Z}_3$  is given by

$$(2.12) \mathcal{Z}_3 = \frac{\tau_{1/(1+\delta\epsilon_1)} \beta L^2 \{ \frac{S}{\alpha\beta} M_2^\alpha M_2^\beta + 2\delta(\alpha\beta) M_2^\alpha M_2^\beta \}}{\beta L^2 - 4\epsilon_1^2 \{ c_{2j}^2 - \frac{\gamma-1}{2} \delta\epsilon_j (\delta\epsilon_j + 2) \frac{h^2}{\eta_j \tau_{1j}} \}}$$

Proof: Putting  $i = 3$  in (2.8), we get

$$(2.13) \mathcal{Z}_3 = \tau_{2/j} L_3^j,$$

which, in consequence of (2.6) of [1], becomes

$$(2.14) \tau_{2/j} (u_{1j}^j - h_{\eta_j} \tau_{1j} \delta\epsilon_j X^j)$$

By virtue of (2.14) and (2.6), we get (2.12).

Theorem 2.3: The gradient of the specific volume  $\mathcal{Z}$  is given by

$$(2.15) \tau_{2/j} = \mathcal{Z}_i M_j^i$$

Proof: From the discussions of section 2, it is obvious that the quantities  $\mathcal{Z}_i$  are completely known. From (2.8) and (2.7) of [1], we, at once, get (2.15).

### 3. GRADIENT OF PRESSURE

We have the following theorems.

Theorem 3.1: The gradient of pressure behind the shock surface is given by

$$(3.1) p_{2/j} = (u_{1j}^j - h_{\eta_j} \tau_{1j} \delta\epsilon_j X^j) S \epsilon_m M_i^l M_j^m.$$

Proof: The equation of motion behind the shock is given by [4]

$$(3.2) u_{2j}^j u_{2/i}^i + \tau_{2/j} p_{2/j} = 0$$

which, in turn, gives

$$(3.3) p_{2/j} = \frac{-u_{2j}^j u_{2/i}^i}{\tau_{2/j}}$$

In consequence of (2.1), (2.4) and (2.19) of [2], the equation

(3.3) easily gives (3.1).

### References

1. Warsi, N. A. (underpublication): On Variation of Velocity and Pressure Behind and Along A Shock Surface in Lagrangian Coordinate System. Savannah State College Faculty Research Bulletin.
2. Warsi, N. A. (underpublication): On Vorticity Behind A Shock Surface in Lagrangian Coordinate System. Savannah State College Faculty Research Bulletin.
3. Warsi, N. A. (1965): Flow Parameters Behind Three Dimensional Shock Wave. Savannah State College Faculty Research Bulletin. Vol. 19, No. 2.
4. Warsi, N. A. (1964): On Geometry of Gas Flow in Lagrangian Coordinate System. Savannah State College Faculty Research Bulletin. Vol. 18, No. 2.
5. Kanwal, R. P. (1959): On Curved Shock Wave in Three Dimensional Gas Flows. Quarterly of Applied Mathematics. Vol. XVI, No. 4.