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Flow Parameters Behind Three Dimensional Shock Wave

by

Nazir A. Warsi

1. INTRODUCTION

The jump conditions for the flow of a perfect gas in Lagrangian Coordinate system is given by [1]

(1.1) $[u^i] + \dot{h}_{n/}[\tau]\, x^i = 0$

(1.2) $[p] + \dot{h}_{n/}[u^i]\, x^i = 0$

(1.3) $[\frac{1}{2} V_{n/}^2 + I] = 0$

In order to determine the flow parameters behind the shock wave we take help of the idea of the shock-strength. The strength of the shock is defined as the ratio of the change in any flow parameter from the backside to the frontside to the flow parameter in front of the shock surface. The idea of shock strength is important from the physical point of view as it determines whether the shock is weak or whether it is to die out soon. As the values of a parameter in two regions approach one another, the discontinuity is removed and the shock dies out. According to the definition of the shock strength if F is a flow parameter, we have

(1.4) $\delta_{F/} = [F]/F_{1/}$

where [F] stands for $F_{2/} - F_{1/}$ In particular the specific volume strength $\delta_{\tau/}$ is given by

(1.5) $\delta_{\tau/} = [\tau]/\tau_{1/}$

2. FLOW PARAMETERS.

We have the following theorems.

THEOREM: 2.1: *The law of conservation of mass in terms of* $\delta_{\tau/}$ *is given by*

(2.1) $[u^i] = -\dot{h}_{n/}\, \delta_{\tau/}\, \tau_{1/}^2\, x^i$

PROOF: In virtue of (1.5), (1.1) gives (2.1).

COROLLARY 2.1: *The equations (2.1) are equivalent to*

(2.2)a $[U_{n/}] = -\dot{h}_{n/}\, \delta_{\tau/}\, \tau_{1/}$

or

(2.2)b $[U_{n/}] = \delta_{\tau/}\, V_{1n/}$

or

(2.2)c $[U_i] = \delta\tau/ \, V_{1/i} \, X^i$

and

(2.3) $[U_\alpha] = 0$

PROOF: Multiplying (2.1) by X^i and summing with respect to i, we get (2.2)a. With the help of the relation $-\dot{h}_{n/} \tau_{1/} = V_{1n/}$, equation (2.2)a gives (2.2)b which is evidently equivalent to (2.2)c.

Multiplying (2.1) by $x^i_{,\alpha}$ summing with respect to i and using the fact that

(2.4) $x^i_{,\alpha} X^i = 0,$

we get (2.3).

COROLLARY 2.2: *For a stationary shock wave, we have*

(2.5) $[U_{n/}] = \delta\tau/ \, U_{1n/}$

PROOF: For a stationary shock wave $\overline{U}_{n/} = 0$. Therefore, putting $V_{\alpha n/} = U_{\alpha n/}$ and $V_{\alpha/i} = U_{\alpha/i}$ in (2.3)b, we get (2.5).

THEOREM 3.2: *The law of conservation of momentum can be put in the form*

(2.6)a $[p] = \dot{h}_{n/} \, \delta\tau/ \, \tau_{1/}$

or

(2.6)b $[p] = -\dot{h}_{n/} \, \delta\tau/ \, V_{1n/}$

or

(2.6)c $[p] = -\dot{h}_{n/} \, \delta\tau/ \, V_{1/i} \, X^i$

which, for the stationary shock, reduces to

(2.7)a $[p] = -\dot{h}_{n/} \, \delta\tau/ \, U_{1n/}$

or

(2.7)b $[p] = -\dot{h}_{n/} \, \delta\tau/ \, U_{1/i} \, X^i$

PROOF: Multiplying (1.5) by X^i and summing with respect to i, we get

(2.8) $[p] + \dot{h}_{n/} \, [U_{n/}] = 0$

which, in consequence of (2.2)a gives (2.6)a. Equation (2.8) also gives (2.6)b if the value of $[U_{n/}]$ is substituted from (2.2)b. Obviously, (2.6)b is equivalent to (2.6)c.

For a stationary shock, $V_{1n/} = U_{1n/}$ and $V_{1/i} = U_{1/i}$

Hence, (2.6)b and (2.6)c reduce to (2.7)a and (2.7)b respectively.

THEOREM 3.3: *The law of conservation of energy at the shock surface can be put in the form*

(2.9)a $[I] = -\frac{1}{2} \, \dot{h}_{n/}^2 \, \delta\tau/ \, (\delta\tau/ + 2) \, \tau_{1/}^2$

or

(2.9)b $[I] = -\frac{1}{2} \, \delta\tau/ \, (\delta\tau/ + 2) \, V_{1n/}^2$

which, for the stationary shock, reduces to

(2.10) $[I] = -\frac{1}{2}\,\delta_{\tau/}\,(\delta_{\tau/}+2)\,U_{1n/}^{2}$

PROOF: (1.3) gives (2.9)a if (1.5) is used. The relation $-h_{n/}\tau_{1/} = V_{1n/}$ reduces (2.9)a to (2.9)b. For a stationary shock $V_{1n/} = U_{1n/}$. Hence, (2.10) is obvious from (2.9)b.

THEOREM 3.4: *The specific volumes on the two sides of the shock surface are related by the equation*

(2.11)a $[\tau] = -\frac{\delta_{\tau/}}{h_{n/}}\,V_{1n/}$

or

(2.11)b $[\tau] = -\frac{\delta_{\tau/}}{h_{n/}}\,V_{1/i}\,\dot{X}^{i}$

which, in the case of a stationary shock, reduce to

(2.12)a $[\tau] = -\frac{\delta_{\tau/}}{h_{n/}}\,U_{1n/}$

or

(2.12)b $[\tau] = -\frac{\delta_{\tau/}}{h_{n/}}\,U_{1/i}\,\dot{X}^{i}$

PROOF: Substituting $\tau_{1/} = -V_{1n/}/\dot{h}_{n/}$ in (1.5), we get (2.11)a which is obviously equivalent to (2.11)b. For a stationary shock $V_{1n/} = U_{1n/}$ and $V_{1/i} = U_{1/i}$. Hence, we get (2.12)a and (2.12)b.

THEOREM 3.5: *In the case of unsteady flow of a polytropic gas* $\delta_{\tau/}$ *is given by,*

(2.13) $\delta_{\tau/} = \frac{2}{\gamma+1}\;\frac{c_{1/}^{2} - \dot{h}_{n/}\,\tau_{1/}^{2}}{h_{n/}^{2}\,\tau_{1/}^{2}}$

PROOF: In the case of a polytropic gas, the energy and specific enthalpy equations are written as

(2.14) $e_{\alpha/} = p_{\alpha/}\,\tau_{\alpha/}/(\gamma-1)$

and

(2.15) $I_{\alpha/} = e_{\alpha/} + p_{\alpha/}\,\tau_{\alpha/}$

respectively. (2.15), by virtue of (2.14), gives

(2.16) $I_{\alpha/} = \frac{\gamma}{\gamma-1}\,p_{\alpha/}\,\tau_{\alpha/}$,

which gives

(2.17) $[I] = \frac{\gamma}{\gamma-1}\,[p\tau]$.

If $\tau_{2/}$, $p_{2/}$ and [I] are eliminated from (2.17) with the help of (1.5), (2.6)a and (2.9)a, then an equation containing $\delta_{\tau/}$ and the flow and thermodynamic parameters of region $1/$, is obtained. This equation gives the value of $\delta_{\tau/}$ as shown in (2.13).

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1. Warsi, N. A. (1965) On Geometry of Shock Waves in Lagrangian Coordinate System. Savannah State College Faculty Research Bulletin. Vol. 18, No. 2.

2. Mishra, R. S. On Stream Lines With Reference to a Shock Surface. Proceedings of National Institute of Sciences. Vol. 26, No 6.

Thermodynamic Parameters Behind Three Dimensional Shock Wave

by

Nazir A. Warsi

1. INTRODUCTION.

The author has determined the flow parameters behind three dimensional shock wave using Lagrangian Coordinate System [1]. The object of this paper is to study the thermodynamic parameters using the same techniques.

2. THERMODYNAMIC PARAMETERS.

We have the following theorems.

THEOREM 2.1: *The sound velocities of the fluid, behind and in front of the shock surface, are related by the equation*

(2.1)a $[c^2] = -\frac{\gamma-1}{2}\delta_{\tau/}(\delta_{\tau/}+2)\dot{h}'^{\,2}_{n/}\tau_{1/}$

or

(2.1)b $[c^2] = -\frac{\gamma-1}{2}\delta_{\tau/}(\delta_{\tau/}+2)V_{1n/}^2$

which, for the stationary shock, reduces to

(2.2) $[c^2] = -\frac{\gamma-1}{2}\delta_{\tau/}(\delta_{\tau/}+2)u_{1n/}^2$

PROOF: The sound velocities in two regions are related by

(2.3) $[c^2] = \gamma[p\tau]$

The pressure and specific volume behind the shock surface are given by [1]

(2.4) $\delta_{\tau/} = [\tau]/\tau_{1/}$

and

(2.5) $[p] = -\dot{h}_{n/}^2\delta_{\tau/}\tau_{1/}$

Substituting the value of $\tau_{2/}$ and $p_{2/}$ from (2.4) and (2.5) in (2.3), we get (2.1)a which, in consequence of the relation $-\dot{h}'_{n/}\tau_{1/} = V_{1n/}$ gives (2.1)b. For stationary shock $V_{1n/} = U_{1n/}$. Hence, we get (2.2).

THEOREM 2.2: *The Mach number behind the shock surface is given by*

(2.6)a $M_{2n/} = \frac{u_{1n/} - \delta_{\tau/}\dot{h}_{n/}\tau_{1/}}{\sqrt{c_{1/}^2 - \frac{\gamma-1}{2}\delta_{\tau/}(\delta_{\tau/}+2)\dot{h}_{n/}^{2}\tau_{1/}^{2}}}$

or

(2.6)b $M_{2n/} = \sqrt{\frac{2}{\tau_{1/}}}\,\frac{u_{1n/} - \delta_{\tau/}\dot{h}_{n/}\tau_{1/}}{\sqrt{2\gamma p_{1/} - (\gamma-1)\delta_{\tau/}(\delta_{\tau/}+2)\dot{h}_{n/}^{2}\tau_{1/}^{2}}}$

or

(2.6)c $M_{2n/} = \frac{u_{1n/} + \delta_{\tau/}V_{1n/}}{\sqrt{c_{1/}^2 - \frac{\gamma-1}{2}\delta_{\tau/}(\delta_{\tau/}+2)V_{1n/}^{2}}}$

PROOF: From the definition of Mach number, we have

(2.7) $M_{2n/} = u_{2n/}/c_{2/}$

Also, the velocity behind the shock is given by [1]

(2.8) $[\dot{u}_{n/}] = -\hbar_{n/}\,\delta_{z/}\,\tau_{1/}$

If we substitute the value of $C_{2/}$, $U_{2n/}$ from (2.1)a, (2.8) in (2.7), we get (2.6)a which in turn gives (2.6)b in virtue of the relation $c_{1/}^{2} = \gamma\, p_{1/}\,\tau_{1/}$. Using the relation $-\hbar_{n/}\tau_{1/} = V_{1n/}$ and equation (2.6)a, we easily obtain (2.6)c.

THEOREM 2.3: *The components of obliquity behind the shock surface are given by*

(2.9) $\Psi_{2/\alpha} = \dfrac{u_{1/i}\, x^{i}_{,\alpha}}{u_{1n/} - \dot{\hbar}_{n/}\,\tau_{1/}\,\delta_{z/}}$.

PROOF: The components of obliquity in a region $\beta/$ is given by

(2.10) $\Psi_{\beta/\alpha} = \dfrac{u_{\beta/i}\, x^{i}_{,\alpha}}{u_{\beta n/}}$

whence, we have

(2.11) $\Psi_{2/\alpha} = \dfrac{u_{2/i}\, x^{i}_{,\alpha}}{u_{2n/}}$.

Substituting for $u_{2/i}$ and $u_{2n/}$ and applying the fact that

(2.12) $X_{i}\, x^{i}_{;\alpha} = 0$,

we get (2.9).

THEOREM 2.4: *For a stationary shock wave, we have*

(2.13) $\dfrac{1}{\delta_{z/}} + \dfrac{1}{\delta_{\psi/}} = 1$

where $\delta_{\psi/}$ *the obliquity strength of the shock wave is defined as*

(2.14) $[\Psi_{\alpha}] = \delta_{\varphi/}/\Psi_{1/\alpha}$

PROOF: Since $-\hbar'_{n/}\tau_{1/} = V_{1n/}$ and $V_{2n/} = U_{1n/}$ for a stationary shock, (2.9) can be written as

(2.15) $\Psi_{2/\alpha} = \dfrac{u_{1/i}\, x^{i}_{,\alpha}}{u_{1n/}\,(1+\delta_{z/})}$

Now, putting $\beta = 1$ in (2.10), we get

(2.16) $\Psi_{1/\beta} = u_{1/i}\, x^{i}_{;\alpha}/u_{1n/}$

This and (2.15) give

(2.17) $\Psi_{2/\alpha} = \Psi_{1/\alpha}\,/\,(1+\delta_{z/})$

which, with the help of (2.14), gives (2.13).

THEOREM 2.5: *The obliquities behind and in front of the shock surface are related by the equation*

$$(2.18)\quad \left[\frac{1}{\psi_\alpha}\right] = -\frac{h'_{n/}\,\delta_{z/}\,\tau_{1/}}{u_{1/i}\,X^{i}_{;\alpha}}$$

PROOF: In consequence of (2.9) and (2.16), we get (2.18).

THEOREM 2.6: *If* e *be the specific internal energy of a polytropic gas, then we have*

$$(2.19)\quad [e] = -\frac{1}{2\gamma}\,\delta_{z/}(\delta_{z/}+2)\,h'^{2}_{n/}\,\tau^{2}_{1/}$$

PROOF: For a polytropic gas, we have

$$(2.20)\quad I_{\alpha/} = e_{\alpha/} + p_{\alpha/}\,\tau_{\alpha/}$$

whence, we get

$$(2.21)\text{a}\quad [I] = [e] + [p\,\tau]$$

or

$$(2.21)\text{b}\quad [I] = [e] + \frac{1}{\gamma}[c^2]$$

But, $[I]$ is given by $[I]$

$$(2.22)\quad [I] = -\frac{1}{2}\,h'_{n/}\,\delta_{z/}(\delta_{z/}+2)\,\tau^{2}_{1/}$$

which, in consequence of (2.1)a and (2.21)b gives (2.19).

THEOREM 2.7: *Specific entropies of a polytropic gas behind and in front of the shock surface are related by the equation*

$$(2.23).\quad [\eta] = JC_v \log \frac{(\delta_{z/})^{\gamma-1}}{2c^{2}_{1/}}\left\{2c^{2}_{1/} - (\gamma-1)\,\delta_{z/}(\delta_{z/}+2)\,h'^{2}_{n/}\,\tau^{2}_{1/}\right\}$$

PROOF: For a polytropic gas [2], we have

$$(2.24)\text{a}\quad \eta_{\alpha/} = JC_v \log p_{\alpha/}\,\tau^{\gamma}_{\alpha/}$$

or

$$(2.25)\text{b}\quad \eta_{\alpha/} = JC_v \log \frac{c^{2}_{\alpha/}\,\tau^{\gamma-1}_{\alpha/}}{\gamma}$$

whence, we get

$$(2.26)\quad [\eta] = JC_v \log \frac{c^{2}_{2/}}{c^{2}_{1/}}\left(\frac{\tau_{2/}}{\tau_{1/}}\right)^{\gamma-1}$$

If we substitute the value of $\tau_{2/}$ from (2.4) and $c^{2}_{2/}$ from (2.1)a in (2.26), we readily get (2.23).

THEOREM 2.8: *The temperature of a polytropic gas behind and in front of a shock surface are related by the equation*

$$(2.27)\quad [T] = -\frac{(\gamma-1)}{2\gamma R}\,\delta_{z/}(\delta_{z/}+2)\,h'^{2}_{n/}\,\tau^{2}_{1/}$$

PROOF: For a polytropic gas, the temperature in a region of the fluid is given by

(2.28) $C_{\alpha/}^2 = \gamma R T_{\alpha/}$

which gives

(2.29) $[c^2] = \gamma R [T]$

Equation (2.29), by virtue of (2.1)a becomes (2.27).

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Deflection of Streams Behind a Curved Shock Wave

by

Nazir A. Warsi

1. INTRODUCTION.

If the angle between the tangent to the stream line and the unit normal vector (X^i) to the shock surface be θ, then

(1.1) $V_{\alpha n/} = V_{\alpha/i} X^i = V_{\alpha/} \cos\theta_{\alpha/}$

where $V_{\alpha/i}$ is the velocity vector in the region $\alpha/$. If the space components of a vector field tangential to the surface are t_i, then

(1.2) $V_{\alpha/i} t_{\alpha/}^i = V_{\alpha/} \sin\theta_{\alpha/}$

The law of conservation of mass at the shock surface is given by [1]

(1.3) $[V^i] = \delta_{z/} V_{1n/} X^i$

Multiplying it by t^i and summing with respect to i we get

(1.4)a $[V^i] t_i = 0$

or

(1.4)b $V_{2/} \sin\theta_{2/} = V_{1/} \sin\theta_{1/} = \mathcal{D}$

2. DEFLECTION OF STREAMS.

We have the following theorems.

THEOREM 2.1: *The angle that the stream line makes with the unit normal* X^i *is given by*

(2.1) $\cot\theta_{2/} = (\delta_{z/} + 1) \cot\theta_{1/}$

PROOF: Multiplying (1.3) by X^i and summing with respect to i, we get

(2.2)a $[V_{n/}] = \delta_{z/} V_{1n/}$

or

(2.2)b $V_{2/} \cos\theta_{2/} = (\delta_{z/} + 1) V_{1/} \cos\theta_{1/}$

Equations (1.4)b and (2.2)b give (2.1).

THEOREM 2.2: *For both the regions, the ratio* $\cot\theta/\tau$ *is constant, that is*

(2.3) $\dfrac{\cot\theta_{\alpha/}}{\tau_{\alpha/}} = \dfrac{1}{\mu}$

PROOF: By virtue of the relation $\delta_{z/} = \frac{[z]}{z_{1/}}$, equation (2.1) gives (2.3).

THEOREM 2.3: *For an unsteady flow behind the shock wave, we have*

(2.4) $\nu = -\mu \, h' n /$

PROOF: Dividing (2.2)b by (1.4)b, we get

(2.5)a $\cot \theta_{2/} = \frac{(\delta_{z/}+1)}{\nu} V_{1/} \cos \theta_{1/}$

or

(2.5)b $\frac{\cot \theta_{2/}}{z_{2/}} = \frac{1}{\nu z_{1/}} V_{1n/}$

which, in consequence of (2.3) and the relation $\delta_{z/} = [z]/z_{1/}$, gives (2.4)

THEOREM 2.4: *The specific volume strength of the shock is defined as the ratio of difference of cotangent of the angle of emergence and the cotangent of the angle of incidence to the cotangent of angle of incidence.*

PROOF: From (2.1), it is obvious that

(2.6) $\delta_{z/} = [\cot \theta] / \cot \theta_{1/}$

3. MAXIMUM DEFLECTION.

Mishra (1960) studied the deflection and found that $\theta_{2/} > \theta_{1/}$ Angle of deflection of the stream behind the shock is given by $[\theta]$ Therefore, the angle of the deflection, Ω is given by

(3.1) $\cot \Omega = \cot[\theta] = \frac{\cot \theta_{1/} \cot \theta_{2/} + 1}{\cot \theta_{1/} - \cot \theta_{2/}}$

Substituting for $\cot \theta_{2/}$ from (2.1), the equation (3.1) gives

(3.2)a $\cot \Omega = \frac{(\delta_{z/}+1) \cot^2 \theta_{1/} + 1}{-\delta_{z/} \cot \theta_{1/}}$

or

(3.2)b $\cot \Omega = \frac{2}{\delta_{z/}} \left(\frac{1 + \delta_{z/} \cos^2 \theta_{1/}}{\sin 2\theta_{1/}} \right)$

or

(3.2)c $\cot \Omega = \frac{\delta_{z/} + 1 + \xi^2}{-\delta_{z/} \xi}$

where $\xi = \tan \theta_{1/}$

Hence, we have the following theorems.

THEOREM 3.1: *In the case of maximum deflection for a fixed $\delta_{z/}$, we have*

(3.3) $\xi = \tan^2 \theta_{1/} = 1 + \delta_{z/}$

PROOF: For the maximum deflection, we have

(3.4) $\frac{\partial \Omega}{\partial \xi} = 0$

Differentiating (3.2)c with respect to ξ and using (3.4), we get (3.3)

THEOREM 3.2. *In the case of the maximum deflection for a fixed* δz_1 *, the ratio of specific volume of two regions is the same as the ratio of square of the tangent of the angle of incidence to unity.*

PROOF: For the maximum deflection, we have

(3.5)a $\tan^2\theta_{1/} = 1+\delta z_1$

or

(3.5)b $\tan^2\theta_{1/} : 1 = z_{2/} : z_{1/}$

THEOREM 3.3. *Maximum deflection for a fixed* δz_1 *is given by*

(3.6) $\cot \underline{\Omega} = \pm \frac{2}{\delta z_1}\sqrt{\delta z_1 + 1}$

PROOF: In consequence of (3.3), the equation (3.2)c gives (3.6).

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