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HOWARD JORDAN, JR., *President*

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Table of Contents

Table of Contents – (Cont'd.)

Mathematics in the Renaissance

By

W. M. Perel

The first observation about mathematics during this period is likely to be that there was not very much done in mathematics. Indeed, mathematics did not flourish during the Roman Empire and probably fared worse in the early Christian period which followed. In books on the history of mathematics, very little attention is devoted to the period from 212 B.C. (the death of Archimedes) to the seventeenth cenutry (the century which produced analytic geometry and the calculus). In *Men of Mathematics* by E. T. Bell, the lives and works of more than forty mathematicians are discussed, but no name is mentioned between Archimedes and Descartes, a span of about seventeen hundred years[1].

While it is true that some advances, particularly in algebra, were made by the Arabs and the Hindus during this period, the greatest contribution of the Arab world was the preservation of the knowledge and contributions of the Greeks. Thus, with the dawn of the Renaissance, men of learning were able to discover anew the vast mathematical contribtuions of the Greek culture.

Mathematics and mathematicians felt the strong disapproval of early Christians. In 415 A.D., an enraged Christian mob's tearing apart Hypathia[2] in the streets of Alexandria for teaching "paganism" proves this fact. She was the greatest female mathematician of the ancient world. Further, Saint Augustine's opinion that all mathematicians had made a pact with the devil is well-known.

In the beginning of the Renaissance, it was natural for those interested in mathematics to turn to the Greeks and particularly to those thirteen books which comprise Euclid's *Elements*. Unfortunately, the early reliance on the methods, ideas, and results of the Greeks had some disadvantages as well as advantages. This reliance tended to stifle initiative and to lead to a kind of pedantry which still remains in many universities of the world.

The Greeks had always emphasized geometry at the expense of other branches of mathematics. Books are written to explain why geometry was preferred by the Greeks, so that their number system and algebra were wretched by comparison with those of the Babylonians which preceded them historically. Here, it is important only to mention that Renaissance mathematicians suffered for a time the handicap of the Greek pro-geometry prejudice.

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[1]Bell, E. T., *Men of Mathematics,* Simon and Schuster: New York, 1937.
[2]Eves, Howard, *An Introduction to the History of Mathematics,* p. 162.
[3]Kline, Morris, *Mathematics in Western Culture,* Oxford University Press, 1953, p. 3.

In their efforts in geometry, the Greeks had always reverenced the circle. The Ptolemaic system of the Universe had to use circles. In the preservation of this system, to include ever newer and better observations, more and more circles had to be used. Roughly the system involved planetary motions in circles about imaginary points which were themselves going in circles about still other imaginary points which at last were traveling about the earth in circular orbits[4].

When at last Copernicus[5] (1473-1543) decided to look afresh at the Universe and put the sun, rather than the earth at the center of things, he was so influenced by his study of Greek geometry that he had to insist on the preservation of circular orbits. The Copernican Theory had the purely abstract and academic advantage over the older Ptolemaic Theory that it required a fewer number of circles and was thus simpler. Such a result is near and dear to the heart of any mathematician, ancient or modern.

But by the scientific standard of agreement with observation, the Copernican Theory was much less satisfactory than the Ptolemaic Theory had been. As a result, the Danish astronomer, Tycho Brahe (1546-1601), who had made by far the most and the best observations of his day, totally rejected the Copernican Theory. It was only after Kepler (1571-1630) finally rejected circular orbits in favor of elliptical ones that a heliocentric theory of the Universe became compatible with observations. Of course, the Greeks had also invented the ellipse centuries before.

Galileo (1564-1642) and others during this period ran into trouble by insisting on experimentation, i.e. observation, as opposed to abstract theorizing which was often unrelated to actual physical results. Again the respect for the abstract thinking of the Greeks, particularly Aristotle (384-322 B.C.), led men to refuse to believe what their eyes told them, if it contradicted an Aristotle opinion. Aristotle had postulated that the speed of a falling object was proportional to its weight. It was more than 1800 years before Galileo demonstrated, by actually dropping weights, that the speed of the fall was unrelated to the weight of the object.

The Greek influence also had an unfortunate effect upon the study of equations. What we think of today as a real number was unknown to the Greeks. Instead, the Greeks thought of a line segment. For example, let us consider the quadratic equation. In modern notation such an equation takes the form:

$$x^2 + ax + b = 0.$$

The modern interpretation is that a and b are given numbers and that the letter 'x' represents an "unknown number, which is to be determined. Since Euclid believed only in line segments and not in real numbers, his results had to be confined to what today we would

[4]See Dreyer, J. L. E., *History of the Planetary System from Thales to Kepler,* Cambridge University Press: London, 1906.

[5]Ibid and also Armitage, Angus, *Copernicus,* W. W. Norton & Co.: New York, 1938.

call "positive real numbers." Therefore, Euclid had to distinguish the following special cases, again written in modern notation:

$$x^2 = ax + b; x^2 + ax = b; x^2 + b = ax.$$

Of course, Euclid did not write them that way, but even if he had, his interpretation would have been geometric. In other words, 'a' and 'b' represent given line segments and a line segment called '1' must also be given. 'x' then is regarded as an unknown line segment. Thus, for example, the first equation above is to be regarded as an equality of areas. The Greeks always regarded a product or a "square" as an area and so did many early Renaissance mathematicians. The first equation simply asks that a line segment be found, so that a square having that line segment as a side will equal the sum of the area of a rectangle having that line segment as one side and 'a' as the other side with the area of a rectangle having '1' as one side and 'b' as the other side. Euclid gives constructions for the answer in each of the three cases illustrated above, in Propositions 5, 6, and 7 in Book II of the *Elements*.[6] Of course, Euclid could only allow positive real solutions, so his methods sometimes produced two, sometimes one, and sometimes no solution.

In the Renaissance, mathematicians began to use algebraic methods acquired from the Arabs, who probably produced something like the famous quadratic formula now taught in the ninth grade. The mathematicians of the day also had knowledge of negative numbers, which were considered interesting because they sometimes increased the number of solutions of an equation, but negative numbers were not accepted as being "real" numbers, because no one could conceive of a line as having "negative length."

All mathematicians follow the ancient slogan, "Be Wise, Generalize!" Therefore, having obtained both algebraic and geometric solutions of the quadratic equation, it was natural to turn to the cubic equation, that is the equation which we would today write:

$$x^3 + ax^2 + bx + c = 0.$$

Written in the many different forms possible, a cubic equation could be regarded as an equation involving an equality of volumes. Thus, such an equation was again a geometric problem. Whether or not Euclid attacked cubic equations from a geometric point of view is not known, but, if he did, he could not have given a construction for a solution. We now know that the construction of a cube root is impossible by Euclidean tools, i.e. by straight edge and compass.

In any case, a host of mathematicians in the fifteenth and sixteenth centuries attacked various special cases of cubic equations by some various ingenious algebraic manipulations. It is generally conceded that Tartaglia (ca. 1506-1557) obtained the first general method, which reduced the problem to the solution of two simultaneous

[6]Euclid's *Elements,* Vol. I, Second Edition, Dover Press: New York, 1956, pp. 383-385.

quadratic equations. In contrast with present policy, Tartaglia did not rush to publish his results, but rather sought to keep them secret. He did, however, explain his method to Cardan (or Cardano, 1501-1576) after receiving Cardan's promise to keep his method secret. However, Cardan published the method almost immediately after receiving it. Although Cardan gave Tartaglia credit for the discovery, fate is such that the method is today known as "Cardan's Method" and may be found under that name in any book on the theory of equations.

With our modern terminology and thinking, it would seem very logical to take the next step to the quartic or fourth degree equation. However, a fourth degree equation could not be solved nor even be conceived as a problem because there was no geometric interpretation available for a fourth power.

Not until the seventeenth century, when Descartes (1596-1650) published his book on geometry, was a geometric interpretation given to powers higher than the third. Descartes was able to consider all products and powers as lengths. It is very interesting to point out that the construction of a length equal to the product of two given lengths (if a length equal to 1 is also given) can easily be constructed by Eucliedean means, although Euclid seems not to have been aware of this fact and made no use of it.

But Ferrari (1522-1565), a pupil of Cardan, solved the quartic equation before Descartes was born. What happened was that a mathematician named Da Coi in 1540 proposed a problem to Cardan which led to a quartic equation. Problems and puzzles about numbers were more and more frequently being proposed, many being of the same type to be found occasionally in the newspapers of our day. It was finally discovered that once a certain algebraic skill had been mastered, most of these "puzzles" could be reduced to solving an equation.

These puzzles, many of them very similar to the "word problems" or "story problems" of high school algebra, played a very important part in giving non-geometric interpretations to equations. Of course, the geometric interpretation was still available as a crutch until the quartic equation was encountered.

Not so much the solution of the fourth degree equations (which has little practical value), but Ferrari's ability to conceive of an equation which could have no geometric interpretation or application, must be counted as one of the great achievements of the sixteenth century. It led to a new freedom. Greek mathematics was still to be studied and respected, even as it is today, but no longer would the Greek preference for geometry be allowed to stifle the efforts of mathematicians and scientists. Thus the end of the Renaissance meant leaving behind not only the so-called "Dark Ages," but also leaving behind forever the former reliance on the ancient world.

With the dawn of the seventeenth century, mathematical activity moved from Italy into France, into England, and finally into Germany, which became and probably remains pre-eminent.

As a final word on the mathematics of the Renaissance, it might be well to mention some things which Renaissance mathematicians did not accomplish. They never achieved a satisfactory notation, so their algebra was always more cumbersome than necessary. For example, exponents, as we know them today, were not known until the middle of the seventeenth century. Although Renaissance mathematicians were acquainted with the Hindu-Arabic numeral system and used it for computations, they seemed to disapprove of the system and reported answers in Roman Numerals. Although they discovered complex numbers as solutions of cubic equations, they dubbed them "imaginary" and had the biassed attiude toward them implied by the name which they gave them. Certainly they made no use of complex numbers and developed no theory of them.

But Renaissance mathematicians in first going back to Greek culture and learning and then finally breaking from it followed the pattern of advance which helped to lead the world into the age of science and discovery.