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A Review of Selected Research Pertaining to Problem Solving in Arithmetic in the Elementary Grades

by

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Purpose

The purpose of this paper is to present a review of selected research pertaining to problem solving in arithmetic in the elementary grades.

Definition of Problem Solving

Definitions of problem solving vary. Russel defines it as "all the activties involved in moving from a task or problem to a goal or solution that the individual finds satisfactory." Paraphrasing Russell, one may say that problem solving in arithmetic is all of those activities involved in a problem or task as the individual pupil sees it to a goal or solution which the teacher and pupil find satisfactory. While this definition may be open to criticisms, it is used operationally in this sense in this paper. (27)

Historical Approach to Problem Solving in General

Bolzano, Decartes and Leibnitz wrote on the study of method in problem solving. Reasoning ability, historically, was one of the "faculties of the mind." According to James (1890) reasoning involves both insight and abstraction. Experimental support from an investigation by Bloom and Broder (1950) has been given to the view which holds that "reasoning" is a form of adjustment. The problem situation, according to this position, produces tension or imbalance in the organism which acts to restore equilibrum. In 1943 Robitaille gave a brief history of the development of reasoning tests. (27)

Plan of Organization

This paper is organized around the following aspects of selected research pertaining to problem solving in arithmetic:

1. The nature of the mental processes of a child solving verbal problems.
2. The effect of problem content and language of the problem statement upon pupil performances.
3. The improvement of learning to solve verbal problems from the environment for the problems in the text.
4. The development of functional problem units.

5. Experimental studies of the relative merits of the procedures to be followed in solving verbal problems.
6. Relations between reading of verbal problems and special reading skills.
7. Meaning in problem solving.
8. Factors important to success in problem solving.
9. Diagnostic studies.
10. Sex differences.

The Nature of the Mental Processes of a Child Solving Verbal Problems

Available evidence indicates that the designation of the mental process of a child solving a verbal problem as reasoning is not appropriate for most pupils.

Monroe in 1928 investigated the nature of the mental processes of a child solving verbal problems of which the purpose was to determine how children solve problems. The investigator studied the performances of a large number of seventh grade pupils on a relative simple problem test. He concluded that a large per cent of the pupils did not reason in trying to solve verbal problems. Instead of reasoning, many of them appeared to perform random calculations upon the numbers given. The responses to problems solved correctly seemed to be determined largely by habit. If no irrelevant data were presented, and the problem was stated in familiar terminology, their response was likely to be correct. When the situations were reversed, relatively few pupils appeared to attempt to reason. They either did not attempt to solve it or else gave an incorrect solution. Thus, this investigation by Monroe gives support to the available evidence which indicates that the designation of the mental processes of a child solving a verbal problem as reasoning is not appropriate for most pupils. (25)

A study of children's procedures in the solution of verbal problems was made by Doty in 1940. The investigator sought to determine how fourth- and sixth-grade children solved problems. It was observed that a "high percentage" did not engage in problem solving in the psychological sense. The investigator questioned the value of the usual problem work in arithmetic. (7)

The findings of studies made by Monroe and Engelhart (26), Monroe (25), and Brownell (4) support the conclusion that pupils tend not to reason when they make efforts to solve problems of the usual textbook types.

In 1949 Johnson made a study of the nature of problem solving in arithmetic at the eighth grade level. He investigated the intellectual factors related to problem solving. The findings of his studies showed a high correlation of general vocabulary with success in problem solving. (20)

Lazerte in 1933 studied the development of problem solving ability. He developed a technique for objectifying the process of thinking used by children in solving problems. This was a worthwhile contribution to research techniques for analyzing the mental processes that produced the answer to a problem. Evidences were found of trial and error procedure, and marked individual differences among students in ways of solving problems. He indicated the positive value of training in analysis and in self-criticism. (22)

Van Engen noted that children learn to recognize what process to perform by visualizing what operations are indicated by the words in the problems. (36)

In 1948 Sutherland conducted an investigation of one-step problem patterns and their relation to problem solving in arithmetic. The purpose of the study was to analyze 15,000 verbal problems in arithmetic with respect to the one-step thought pattern involved. With minor exceptions, these problems represented all the verbal problems in four series of widely used arithmetic texts covering grades three to six. His significant findings were as follows:

1. All of the 15,000 problems could be classified under thirty-eight different one-step thought patterns, of which four were in substraction, 10 in addition, 8 in multiplication, and 16 in division.
2. A wide variation existed among the authors of the texts studied in their grade-by-grade treatment of the thought patterns.
3. In light of the total repetitions of the thought patterns in all of the grades combined, there was close agreement.
4. On the basis of the total repetitions of the thought patterns in all four grades combined, the multiplication pattern ranked first in frequency, the division pattern second, the substraction pattern third, and the addition pattern fourth.

The implication of the study is that the ability to solve problems can be improved by letting children become familiar with the various thought patterns.(30)

The Effect of Problem Content and Language of the Problem Statement Upon Pupil Performance

Johnson sought to measure the effect of instruction in mathematical vocabulary. His subjects consisted of 898 pupils in 28 seventh-grade classes taught by fifteen different teachers. A control group and an experimental group were used. He found the following:

1. No reliable difference in any part of the Analytical Scales of Attainment.
2. A reliable difference favoring the experimental group on vocabulary tests containing the words taught them.
3. No difference on a test of transfer in learning vocabulary.

4. Superiority for the experimental groups on two of his specially prepared tests in problem solving.(17)

Buswell reported a study by Brownell and Stretch in which the results were analyzed in relation to the four levels of familiarity and in respect to other factors such as accuracy of computation, choice of arithmetical operations to use, and the effect of each of the following conditions: (1) form of presentation, (2) amount of time, (3) order of presentation, and (4) difficulty of the problems. The conclusion reached was that the correct choice of operation showed a positive relation to a degree of familiarity, but for certain problems this was not the case.(7)

White sought an answer to the question "does experience in the situation involved affect the solving of a problem?" She found a statistically reliable relationship between experience and ability to solve verbal problems. In checking on experience, she actually checked with the pupils. She did not rely upon her judgment.(38)

Research Directed Toward the Improvement of Learning to Solve Verbal Problems

The effect of selected cues in children's solution of verbal problems was investigated by McEwen in 1941. He observed poor results from the use of cues and concluded that teaching cues is not justified.(24) In 1948 Hartung made a study of advances in the teaching of problem solving. He analyzed the nature and difficulties of problem solving in arithmetic. The investigator noted that progress was being made in the area.(14)

The Development of Functional Problem Units

Several studies have been concerned with the development of functional problem units. During the 1944-45 academic year, Upley developed a functional problem unit on the topic "thrift" with a group of fifth grade pupils. The unit of work was a problem closely connected with the interests of the pupils. The pupils were able to see meaning in the unit, because it was related to life. As a result of the development of this functional problem unit, Upley made the following conclusions:

1. The child's experience plays a vital part in achieving the correct answers to problem work.
2. Large numbers are reduced in difficulty when problem work is supplemented by real life situations.
3. Errors are greatly reduced when problem work is presented in meaningful situations.
4. Success of children who have low I.Q.'s can be improved when problem work is vital.
5. Meaningful problem work can be an inspiration for advanced children with a high I.Q.

6. Work on informational units improves judgment and choice of values, not alone in problem work.(35)

Shea in 1944-45 made an experimental study of problem solving, the purpose of which was to develop functional units of problems in the elementary grades. The subjects used in the study were two eighth grade divisions which spent one period a week on the informational problem unit. Many units required a period ranging from a few weeks to several months. The informational problem unit was defined as a unit of work in arithmetic in which a problem closely related to the child's community and interests for the solution of which the child must go, search for and gather the necessary data, information, and facts. The results showed that of the 69 pupils taking part in the unit, 54 favored the informational problem units, nine favored the problems in the text, and six felt that the informational problem units and the written problem units of the text had value. The conclusion reached was that informational problem units can and should take the place of written problems in the text.(28) Similar studies have been made by Dexter (9), Mccann (23), and Stone.(29)

Experimental Studies of the Relative Merits of the Procedures to Be Followed in Solving Verbal Problems

A number of experimental studies of the relative merits of the procedure to be followed in solving verbal problems have been made. Hall in 1942 investigated oral aids to problem solving. He found it effective for his subjects in three grade 5 and grade 6 classes to read, discuss their own problem orally, select the appropriate operations in group work, and estimate the answer before solving.(12)

In 1944 Klugman investigated cooperative versus individual efficiency in proble msolving. He used control groups and experimental groups. Pairs were matched for CA, IS, sex, race, and grade. The findings showed that children who worked together were more successful in solving problems than were control children who worked alone.(21)

An experimental study of two types of arithmetic problems was conducted by Bramhall in 1939, the purpose of which was to determine the comparative merits of "conventional" problems as compared with "imaginative" problems. Seven classes (214 children) spent three class periods a week entirely on conventional problems while the same number of classes (213 children) spent three class periods a week on imaginative problems solving them any way they wished. The investigator was unable to find any difference in the results of exclusive training with either kind according to the measurement made with several different kinds of standard tests after ten weeks.(3)

An earlier study made by Harry Wheat found that children were more successful with the conventional type, while Garry Myer's study showed that children were more successful with the imaginative type.

In 1938 Thiele sought to determine the contribution of generalization to the learning of the addition facts. Three different methods of teaching children to solve problems were compared, namely: (1) the "vocabulary method" in which the subjects selected words from several suggested words that would make sensible problems for the facts given, (2) the "association method" according to which children corrected their errors with a minimum of help after "type or model" solutions were consulted when they were in difficulty, and (3) the "analysis method" in which the usual questions were used to study problems such as "What is given?" "What is to be found?" "Which is the correct solution?" After fifteen weeks of the above instruction, ten minutes a day were spent on the same set of problem exercises. A total of 182 trios of fourth grade subjects were available for comparisons. By reliable differences the "ascociation group" surpassed all other groups. It was concluded that it is better to give children plenty of exercise in problem solving by their own devices than to impose upon them artificial procedures which they can not understand or use effectively.(32)

In 1953, Burch conducted an experimental study of the effectiveness of teaching problem solving techniques in grades 4, 5, and 6. He found that scores made by children on tests not requiring them to go through steps of formal analysis tended to be higher than those in which they were required. The investigator concluded that a more profitable attack included thinking more carefully about size, relationships, and dynamics of quantities described in each problem.(5)

Beatly expressed disapproval of all systems (such as step-analysis) for problem solving. He recommended that teachers "supply" a sufficient number of genuine problems which the pupil is likely to solve successfully by his own thinking combined with enough problems that stretch the pupil's power in order to keep interest.(1)

In 1950, Welch investigated the relative merits of two types of arithmetic problems. Evidence from the study indicated that problems expressing social situations in life were not solved with any greater success than were "un-real" problems and that pupils tended to prefer those which were unrelated to life situations.(37)

Relation Between Reading of Verbal Problems and Special Reading Skills

In 1944 Treacy sought to discover the relationship of reading skills to the ability to solve problems. He found nearly reliable differences in arithmetic vocabulary and four other reading skills. The conclusion reached was that future research on his problem must deal with reading as a single unitary ability.(33)

Hansen in 1944 studied factors associated with successful achievement in problem solving in sixth grade arithmetic. He administered nine arithmetic tests, ten mental tests, and seven reading tests to 681 sixth grade pupils. Evidence from his study suggests that specific reading skills are associated with successful achieevment in problem solving.(13)

Meaning in Problem Solving

In 1948 Cronbach emphasized a psychological approach to problem solving in arithmetic. He suggested that psychological difficulties are likely to be encountered in the problem solving program in arithmetic.(8)

Factors Important to Success in Problem Solving

Sutherland investigated some aspects of problem solving in 1941-42, the purpose of which was two-fold: (1) to determine the effect of familiarity of the situation on pupil's ability to solve arithmetic problems and (2) to ascertain what abilities were involved in the solution of an arithmetic problem by factorial analysis. The findings according to statistical analysis indicated five factors of importance to success. He identified three of these factors as "g," a verbal factor, and a number factor. All of them are apparently equal in potency.(31)

Diagnostic Studies

John made an extensive study of difficulties in solving problems in arithmetic. She made a detailed analysis of the working processes of 60 pupils in solving two-step problems.

The subjects were selected from two schools in which the emphasis on problem solving had been quite different. The five poorest and five best pupils were selected from grades four through six. The investigator listed and described 40 specific types of errors. The study has been recommended as a very useful guide for the diagnosis of difficulties in problem solving in the upper grades.(16)

Sex Differences

Blackwell conducted a comparative investigation into factors involved in the mathematical ability of boys and girls. The purpose of the experiment was to isolate, interpret, and compare the mental components of the mathematical ability of boys and girls in the age range of 13½ to 15 years and to try to estimate their significance by subjecting the results obtained from a battery of tests to factorial analysis. He concluded that certain definite factors were involved in the composite mathematical functioning and that these factors are different in boys and girls. In the case of boys three specific factors "g," "o," and "w" were found to enter into mathematical ability while in the case of girls four mental components "g," "o," "v" and "x" were found. The first factor of importance in the mathematical ability of boys and girls is a common factor which appears to be similar to the "g" of Spearman.(2)

In 1955 Engelhard conducted an experimental study of arithmetic problem solving ability of sixth-grade girls. The purpose was to ascertain, by a comparison of differences between means, the abilities differentiating high-and-low achieving girls in arithmetic problem

solving at the sixth-grade level: and to determine by a comparison of the findings of this study with those of a comparable investigation dealing with boys whether sex differences in problem solving ability exist at this level. The subjects included 1101 boys and girls constituting 27 sixth-grade classes in 22 schools of Milwaukee and its environs. The investigator found that the girls differ from the boys (1) by a higher MA and IQ, but a lower chronological age, (2) by a more favorable attitude toward arithmetic, and (3) by superior achievement, especially on tests of the mental factor and reading factors.

One of the major implications of this study is that while general intelligence influences problem solving ability, it is not the sole, nor dominant factor making for success in arithmetic problem solving. Sixth-grade pupils who were equated on the basis of MA were distinguished as superior and inferior achievers in problem solving by reason of varying degree of proficiency in certain specific abilities.(10)

SUMMARY

The purpose of this paper was to present a review of selected research pertaining to problem solving in arithmetic in the elementary grades. Research studies were examined in the following areas: the nature of the mental processes of a child solving verbal problems, the effect of problem content and language of the problem statement upon pupil performances, the improvement of learning to solve verbal problems from the environment for the problems in the text, the development of functional problem units, experimental studies of the relative merits of the procedures to be followed in solving verbal problems and special reading skills, meaning in problem solving, factors important to success in problem solving, diagnostic studies, and sex differences.

It was seen that the earlier studies of problem solving made by various investigators sought to find better techniques of problem solving. The evidence from most of the studies tends to support the conclusion that experience in solving many problems seems to be the answer.

Some studies have dealth with the nature of the problems to be solved in contrast to the processes employed in solving them. Some of these studies support the conclusion that problems selected by pupils themselves from their environment is of significance in problem solving. Still other studies support the view that having children select problems from their environment is of no significance in problem solving.

In general, the writer of this paper found research in problem solving to be fragmentary, inconsistent, frequently inclusive, lacking in proper controls, and inadequate in sampling. From the point of view of methodological adequacy and the current status of research in problem solving in arithmetic in the elementary grades, it may be seen that there is a need for more standardized experimental situations and a need for specifying the ability of the subjects used.

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